Introduction to Airframe Aeroacoustics Including Applications in Turbulent Flows

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Introdução à Aeroacústica

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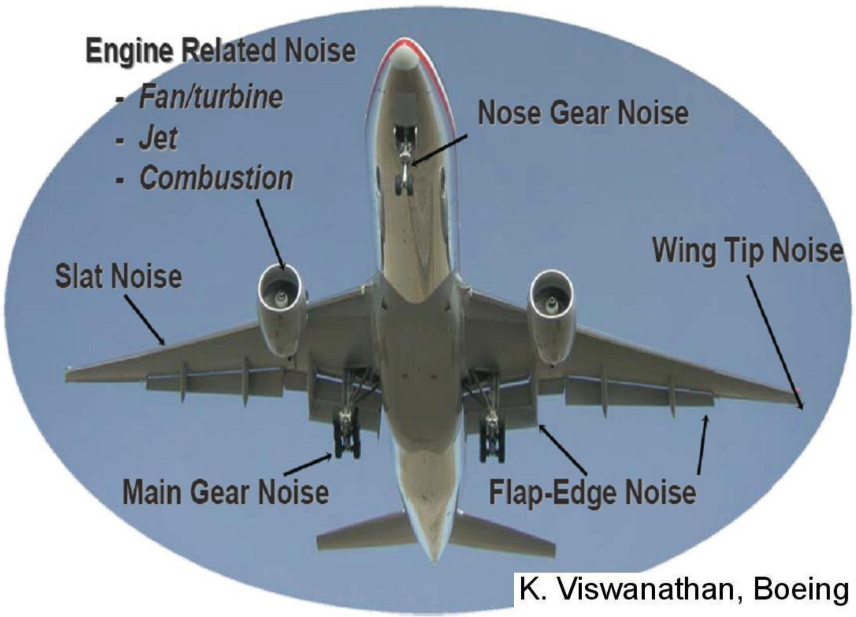
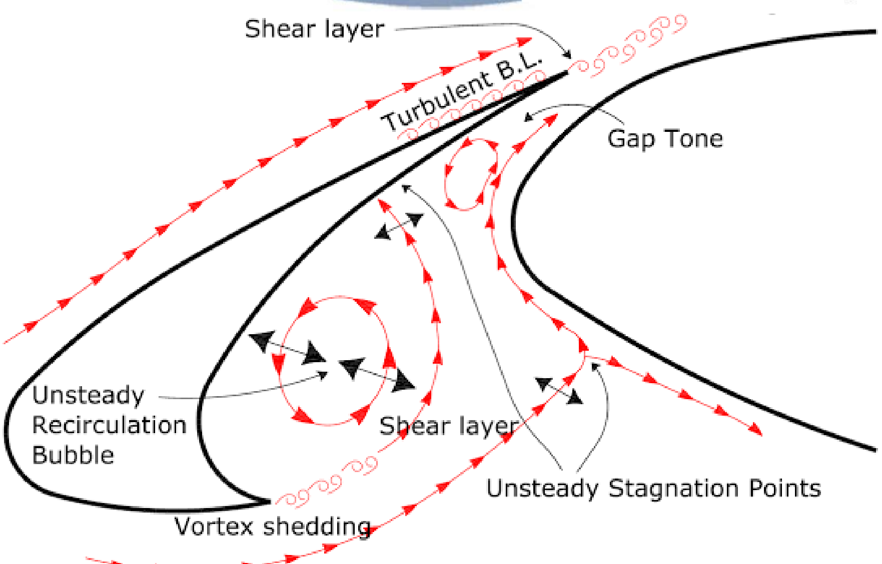
Aeroacoustics

- Aeroacoustics: field of study that deals with sound generation/propagation in unsteady aerodynamic flows
- Pioneered by Sir James Lighthill in the 50's who was interested in jet noise reduction
- Applications: aircraft, automobiles, wind and gas turbines, musical instruments, fans and rotors, home appliances, etc…

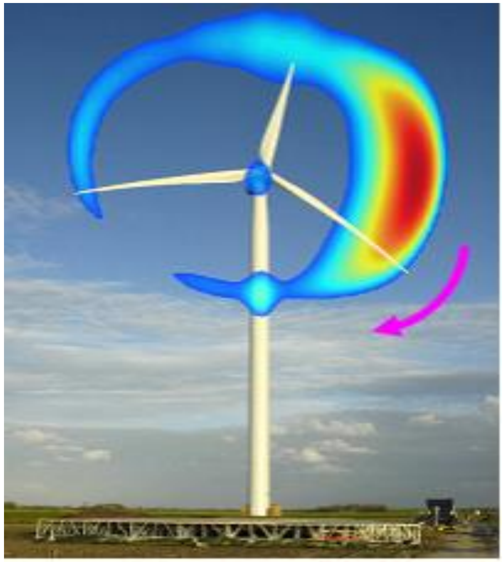
Airframe Noise

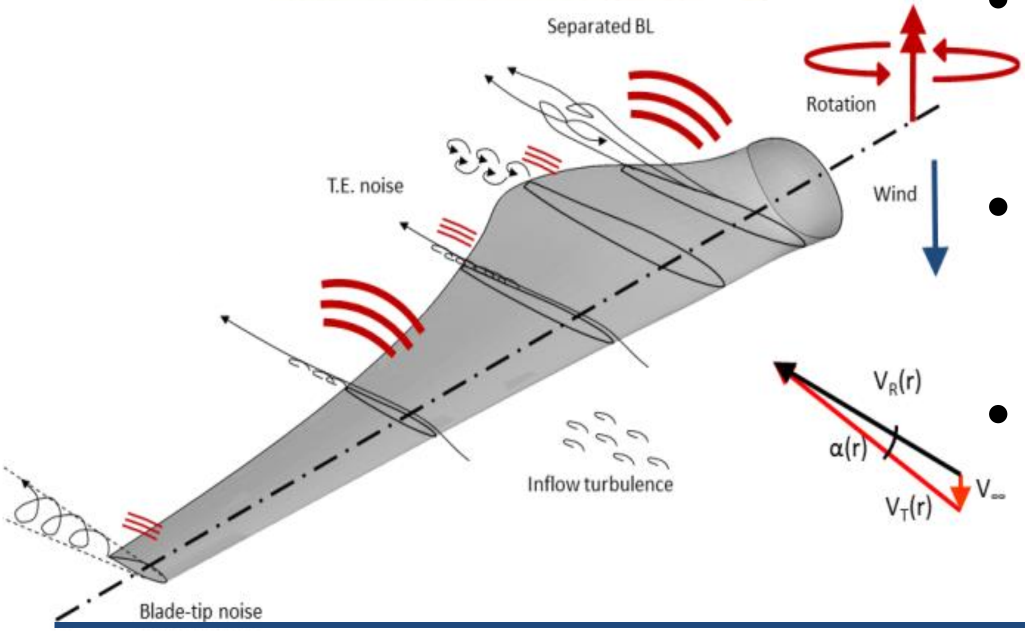




- Jet noise reduction over last decades
- Airframe became an important noise source at landing configuration
- Main sources are high-lift components and landing gears
- Complex flow physics: shear layers, cavities, sharp and blunt bodies, wake interaction, etc…

Wind Turbine Noise

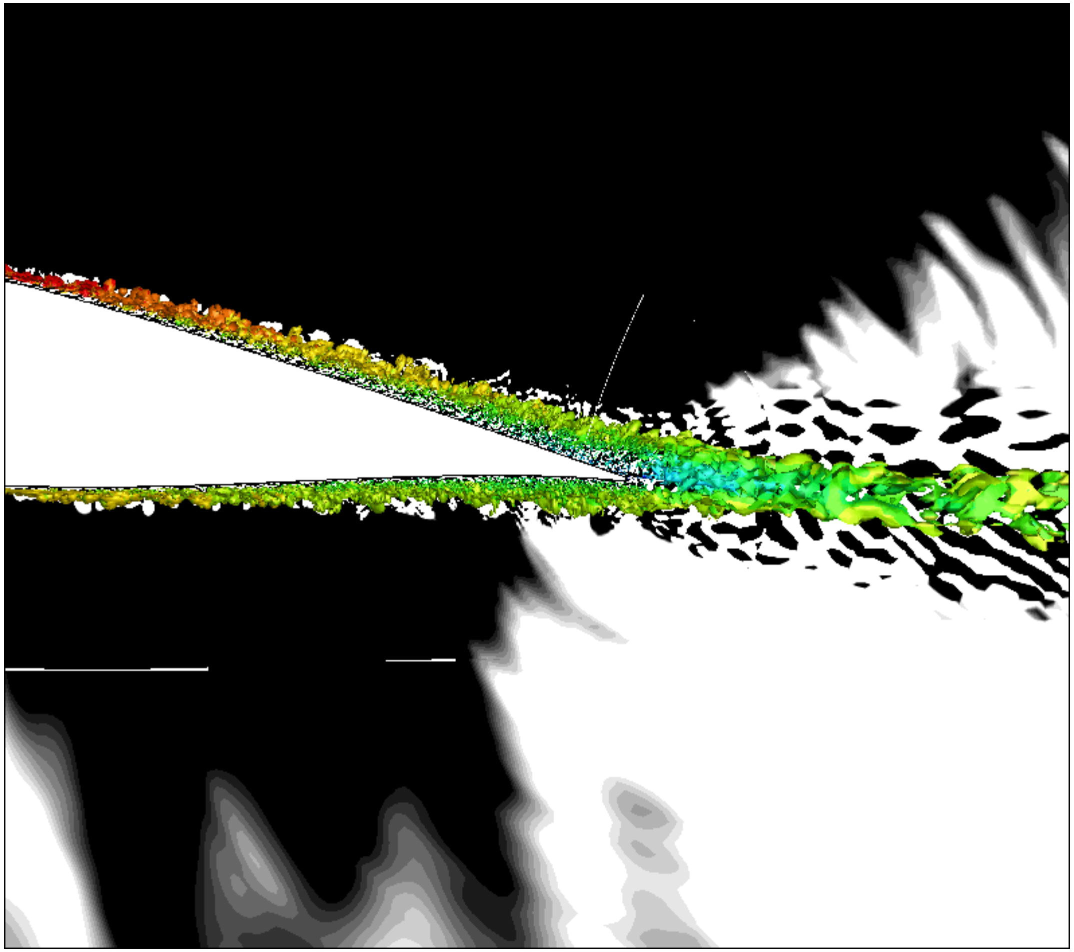




- Regulations on noise generation are more stringent
- Noise is an important factor on wind turbine design
- Airfoil leading-edge noise from inflow turbulence
- Boundary layer trailing-edge noise
- Other sources: stall noise, trailing-edge bluntness noise, etc…

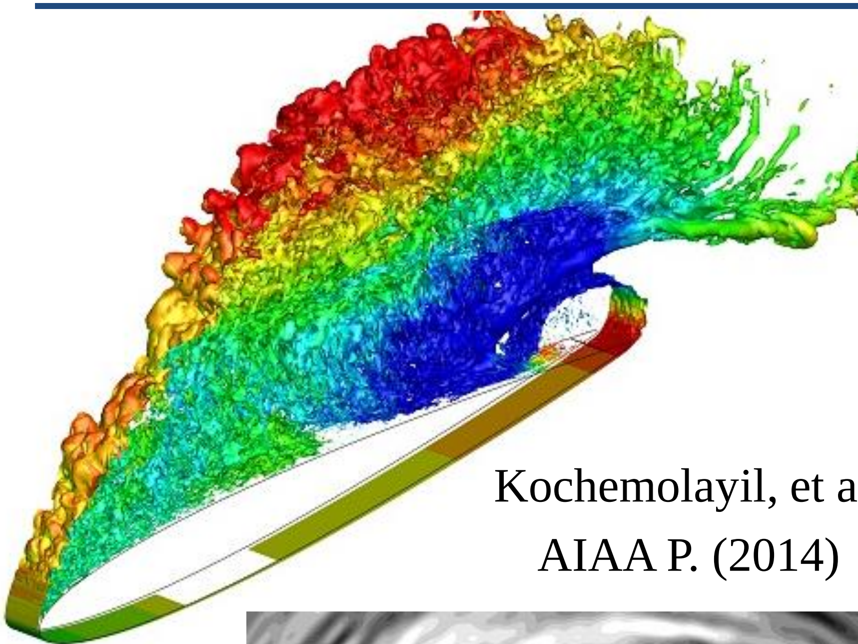
Turbulence and Sound (Noise?)

Wolf et al., AIAA Paper (2012)

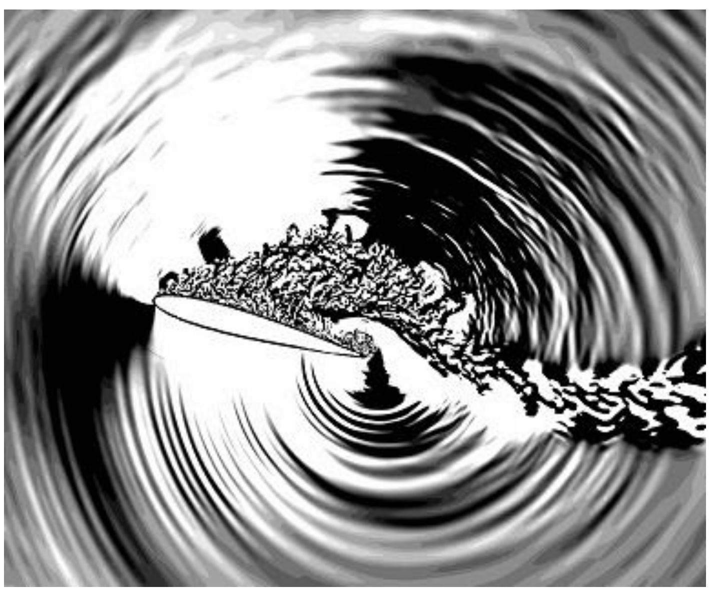


- Sound is a by-product of unsteady fluid motions
- Turbulent velocity fluctuations generate noise
- Conversion of hydrodynamic energy into acoustic energy
- Sound propagates as longitudinal waves that vibrate our eardrums

Outline



Kochemolayil, et al.
AIAA P. (2014)



- From the Navier-Stokes eqs. to the wave equation
- Elementary sources
- Some acoustic analogies
- Landing gear noise
- Airfoil noise
- Poro – elastic trailing edges

Fundamentals

Let us start from the Navier-Stokes equations which model unsteady, viscous, compressible flows

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \qquad \text{Mass}$$

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u} + p\mathbf{I} - \boldsymbol{\sigma}) = 0 \qquad \text{Momentum}$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho \mathbf{u} E + p\mathbf{u} - \boldsymbol{\sigma} \cdot \mathbf{u} + \mathbf{q}) = 0 \qquad \text{Energy}$$

Closure: ideal gas, Stokes' hypothesis, Fourier's law, Gibbs equation

Fundamentals

- For many applications sound is linear
  - absence of non-linear interactions among waves
  - superposition of solutions is allowed
- Viscous effects can be neglected
  - acoustic Reynolds numbers are quite high for most frequencies of interest

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \mathbf{u}' = m' \qquad c_0^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

$$\rho_0 \frac{\partial \mathbf{u}'}{\partial t} + \nabla p' = \mathbf{F} \qquad \rho' = p'/c_0^2$$

Fundamentals

Wave equation for pressure fluctuation

$$\frac{\partial}{\partial t}\left(\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \mathbf{u}'\right) = \frac{\partial m'}{\partial t}$$

$$\nabla \cdot \left(\rho_0 \frac{\partial \mathbf{u}'}{\partial t} + \nabla p'\right) = \nabla \cdot (\mathbf{F})$$

subtract

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) p' = \frac{\partial m'}{\partial t} - \nabla \cdot \mathbf{F}$$

$$\hat{p}'(\mathbf{x},\omega) = \int_{-\infty}^{\infty} p'(\mathbf{x},t) e^{i\omega t} dt$$

Fourier transform

$$\nabla^2 \hat{p}' + k^2 \hat{p}' = i\omega \hat{m}' - \nabla \cdot \hat{\mathbf{F}}$$

Helmholtz equation

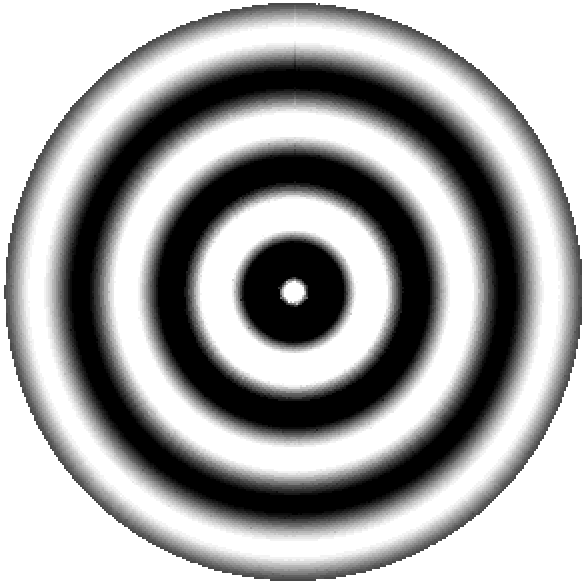
Elementary Sources

In practical problems of engineering, noise sources can be modeled using elementary sources

Monopole sources: mass injection, unsteady heat addition

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = -q(\tau)\delta(\mathbf{x}-\mathbf{y})$$

$$p'(\mathbf{x},t) = -\frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{q(\mathbf{y},t-\frac{||\mathbf{x}-\mathbf{y}||}{c_0})}{||\mathbf{x}-\mathbf{y}||}d^3\mathbf{y}$$



Point monopole

Elementary Sources

Dipole sources: unsteady surface loading

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) p' = \mathbf{F}(\tau, y) = -\nabla \cdot [\mathbf{f}(\tau)\delta(\mathbf{y})]$$

$$p'(\mathbf{x}, t) = \int_{-\infty}^{\infty} \frac{\partial}{\partial y_j} \frac{f_j\left(\mathbf{y}, t - ||\mathbf{x} - \mathbf{y}||/c_0\right)}{4\pi||\mathbf{x} - \mathbf{y}||} d^3\mathbf{y}$$

Since $\partial/\partial y_j = -\partial/\partial x_j$

$$p'(\mathbf{x}, t) = -\frac{\partial}{\partial x_j} \int_{-\infty}^{\infty} \frac{f_j\left(\mathbf{y}, t - ||\mathbf{x} - \mathbf{y}||/c_0\right)}{4\pi||\mathbf{x} - \mathbf{y}||} d^3\mathbf{y}$$

Point dipole

Elementary Sources

For a point dipole one has:

$$\frac{\partial}{\partial x_j}\left(\frac{f_j(\mathbf{y},t-\frac{||\mathbf{x}-\mathbf{y}||}{c_0})}{4\pi||\mathbf{x}-\mathbf{y}||}\right)=\frac{1}{4\pi||\mathbf{x}-\mathbf{y}||}\frac{\partial}{\partial x_j}\left(f_j(\mathbf{y},t-||\mathbf{x}-\mathbf{y}||/c_0)\right)+$$

$$f_j(\mathbf{y},t-||\mathbf{x}-\mathbf{y}||/c_0)\frac{\partial}{\partial x_j}\left(\frac{1}{4\pi||\mathbf{x}-\mathbf{y}||}\right)$$

which finally gives:

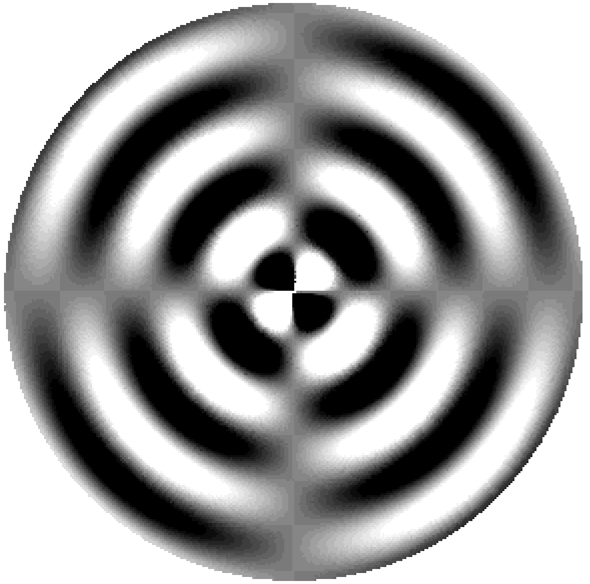
$$p'(\mathbf{x},t)=\frac{cos\theta}{4\pi}\left(\underbrace{\frac{\partial f_j}{\partial t}\frac{1}{c_0||\mathbf{x}-\mathbf{y}||}}_{\text{far-field}}+\underbrace{\frac{f_j}{||\mathbf{x}-\mathbf{y}||^2}}_{\text{near-field}}\right)$$

$cos\theta=(\mathbf{x}-\mathbf{y})/||\mathbf{x}-\mathbf{y}||$ dipoles are directive

Elementary Sources

Quadrupole sources: turbulence

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) p' = \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}(\mathbf{y}, \tau)$$



Point lateral quadrupole

Quadrupole sources are less efficient noise sources (than dipoles and monopoles) due to cancelation

$$p'(\mathbf{x}, t) = \frac{1}{4\pi}\frac{\partial^2}{\partial x_i \partial x_j}\int_{-\infty}^{\infty} \frac{T_{ij}(\mathbf{y}, t - ||\mathbf{x} - \mathbf{y}||/c_0)}{||\mathbf{x} - \mathbf{y}||} d^3\mathbf{y}$$

Lighthill's Acoustic Analogy

- Aerodynamic noise can be predicted by the direct solution of the Navier-Stokes equations
- However, solving a wave equation is easier!
- Lighthill showed that a re-arrangement of the Navier-Stokes equations leads to:

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)(c_0^2\,\rho') = \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}$$

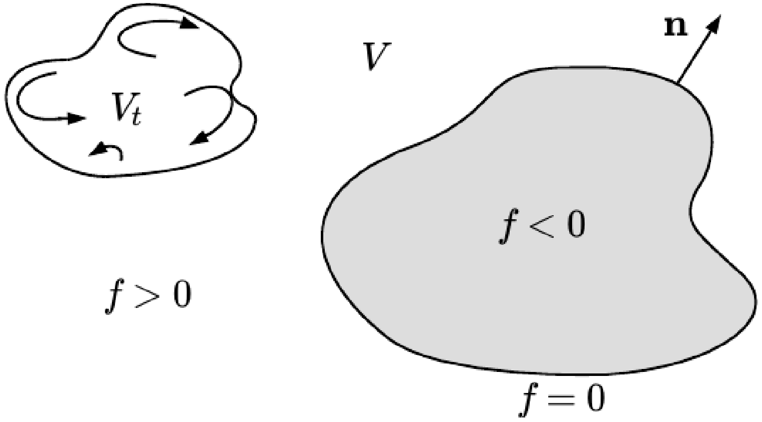
$$T_{ij} = \rho u_i u_j + (p' - c_0^2\rho')\,\delta_{ij} - \sigma_{ij}$$

Lighthill's Acoustic Analogy

- Lighthill's analogy models a complex flow process using an equivalent source
- How useful is this new equation? Is it simpler than solving the Navier-Stokes equations? How can we solve it?
- The Navier-Stokes equations need to be solved anyway through experiments, numerical simulation or using analytical models
- In general, the sound sources are limited to the turbulent flow region

Curle's Analogy

- Acoustic analogies for airframe noise problems
- Curle extended Lighthill's analogy to account for noise generation by solid surfaces immersed in turbulent flows



$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\left(H c_0^2 \rho'\right) =$$

$$\underbrace{\frac{\partial}{\partial t}\left(\rho u_j \frac{\partial H}{\partial x_j}\right)}_{\text{monopole}} - \underbrace{\frac{\partial}{\partial x_i}\left(\left(\rho\, u_i\, u_j + p'_{ij}\right)\frac{\partial H}{\partial x_j}\right)}_{\text{dipole}} + \underbrace{\frac{\partial^2 (H T_{ij})}{\partial x_i \partial x_j}}_{\text{quadrupole}}$$

Curle's Analogy

Solution of Curle's analogy includes volumetric and surface sources

$$H c_0^2 \rho' = \frac{\partial^2}{\partial x_i \partial x_j} \int_V \frac{T_{ij}}{4\pi ||\mathbf{x} - \mathbf{y}||} \, d^3\mathbf{y} + \frac{\partial}{\partial t} \oint_S \frac{\rho \, u_j}{4\pi ||\mathbf{x} - \mathbf{y}||} \, dS_j(\mathbf{y})$$

$$- \frac{\partial}{\partial x_i} \oint_S \frac{\rho \, u_i \, u_j + p\delta_{ij} - \sigma_{ij}}{4\pi ||\mathbf{x} - \mathbf{y}||} \, dS_j(\mathbf{y})$$

Surface can be a solid body immersed in turbulent flow or a "permeable" flow region

Curle's Analogy

- For low Mach number flows, quadrupole sources can be neglected (w.r.t. to dipoles)
- Considering a surface surrounding a solid body

$$Hc_0^2\rho' = -\frac{\partial}{\partial x_j}\oint_S \frac{p'}{4\pi||\mathbf{x}-\mathbf{y}||}\,dS_j(\mathbf{y})$$

- Assuming a compact body (L << λ) and a far-field observer

$$c_0^2\rho'(\mathbf{x},t) \approx \frac{-x_j}{4\pi c_0||\mathbf{x}||^2}\frac{\partial}{\partial t}\oint_S p'\left(\mathbf{y},\, t-\frac{||\mathbf{x}||}{c_0}+\frac{\mathbf{x}\cdot\mathbf{y}}{c_0||\mathbf{x}||}\right)n_j dS(\mathbf{y})$$

Ffowcs Williams and Hawkings Analogy

- FW&H extended Curle's analogy: observers, sources and control surfaces can move

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2\nabla^2\right)(H\rho') = \frac{\partial^2 (HT_{ij})}{\partial x_i \partial x_j} + \frac{\partial}{\partial t}\left\{[\rho(u_i - v_i) + \rho_0 v_i]\frac{\partial f}{\partial x_i}\delta(f)\right\} -$$

FW&H equation

$$\frac{\partial}{\partial x_i}\left\{[\rho u_i(u_j - v_j) + p\delta_{ij} - \sigma_{ij}]\frac{\partial f}{\partial x_j}\delta(f)\right\}$$

$$H c_0^2\,\rho'(\mathbf{x},t) = H\,p'(\mathbf{x},t) = \frac{\partial^2}{\partial x_i \partial x_j}\int_V \frac{HT_{ij}\delta(t-\tau-||\mathbf{x}-\mathbf{y}||/c_0)}{4\pi||\mathbf{x}-\mathbf{y}||}\,d^3\mathbf{y}\,d\tau -$$

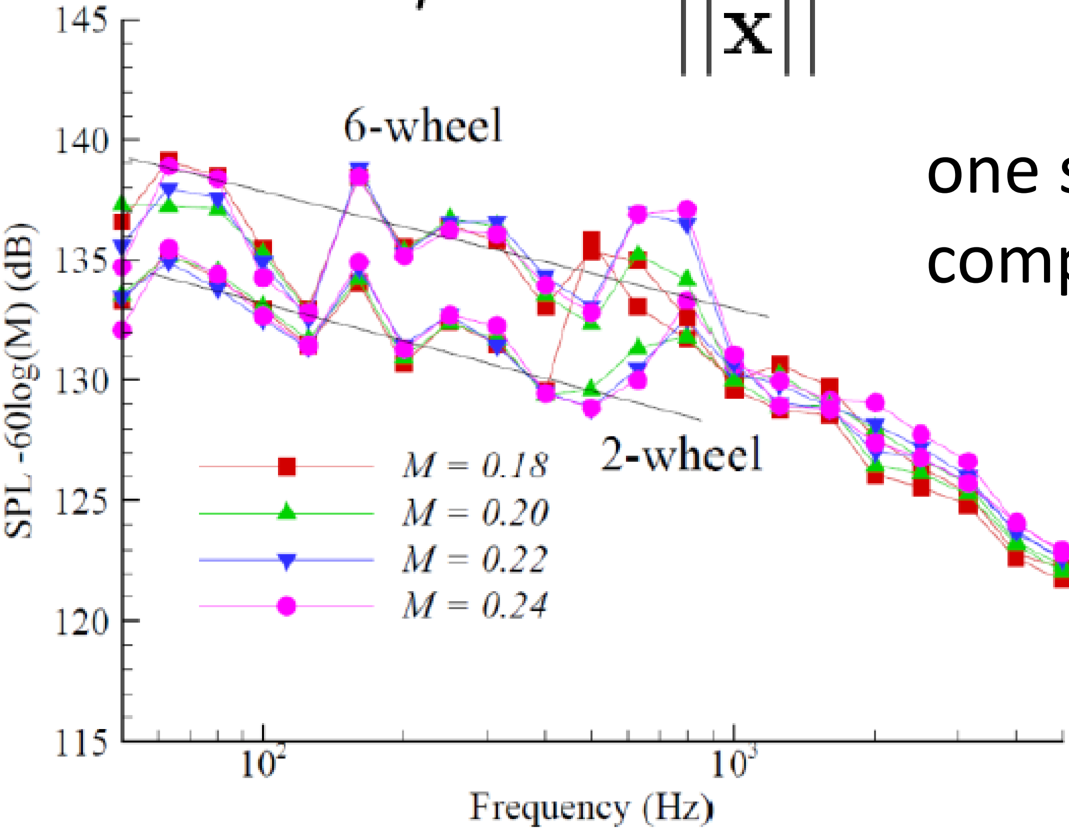
$$\frac{\partial}{\partial x_i}\int_V \frac{[\rho u_i(u_j - v_j) + p'\delta_{ij} - \sigma_{ij}]}{4\pi||\mathbf{x}-\mathbf{y}||}\frac{\partial f}{\partial x_j}\delta(f)\delta\left(t-\tau-\frac{||\mathbf{x}-\mathbf{y}||}{c_0}\right)d^3\mathbf{y}\,d\tau +$$

$$\frac{\partial}{\partial t}\int_V \frac{[\rho(u_j - v_j) + \rho_0 v_j]}{4\pi||\mathbf{x}-\mathbf{y}||}\frac{\partial f}{\partial x_j}\delta(f)\delta\left(t-\tau-\frac{||\mathbf{x}-\mathbf{y}||}{c_0}\right)d^3\mathbf{y}\,d\tau$$

Curle's Analogy

From Curle's analogy, for compact bodies

$$\rho' \sim \frac{\rho_0 M^3 L}{||\mathbf{x}||} \quad \Rightarrow \quad \bar{p}'^2 \sim M^6$$



one should remind that for compact quadrupoles $\bar{p}'^2 \sim M^8$

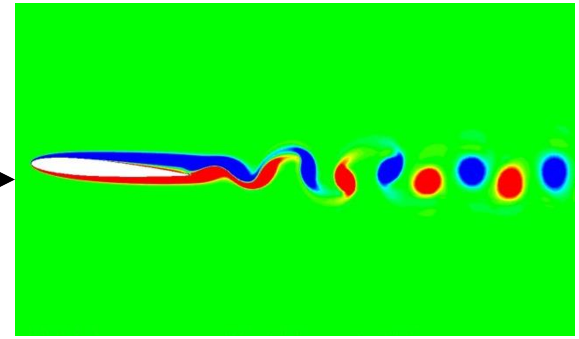
Results from Guo *et al.* (2004) for noise scaling of 2- and 6-wheel landing gears

Hybrid Method for Noise Prediction

High-fidelity CFD

Need for non-dissipative/non-dispersive numerical methods

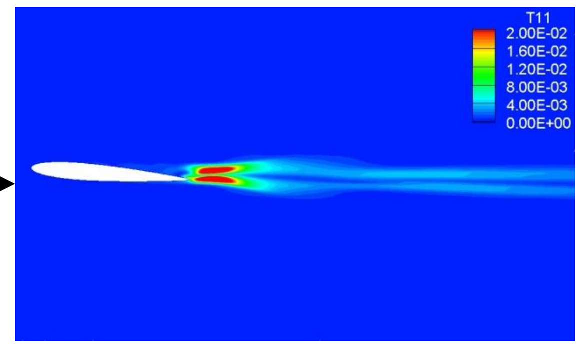
Flow field



Post-processing

Periodic? Intermittent? FFT, wavelets, POD?

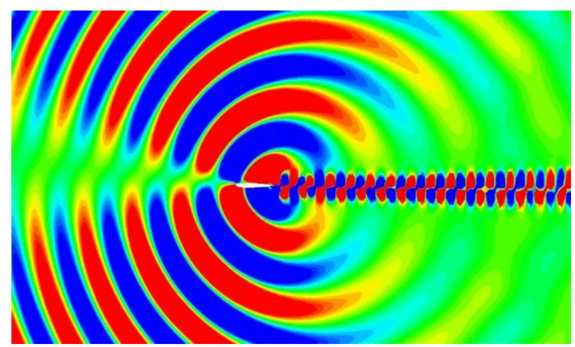
Acoustic sources



Acoustic analogy

Wave equation, non-linear Euler equations?

Far field sound

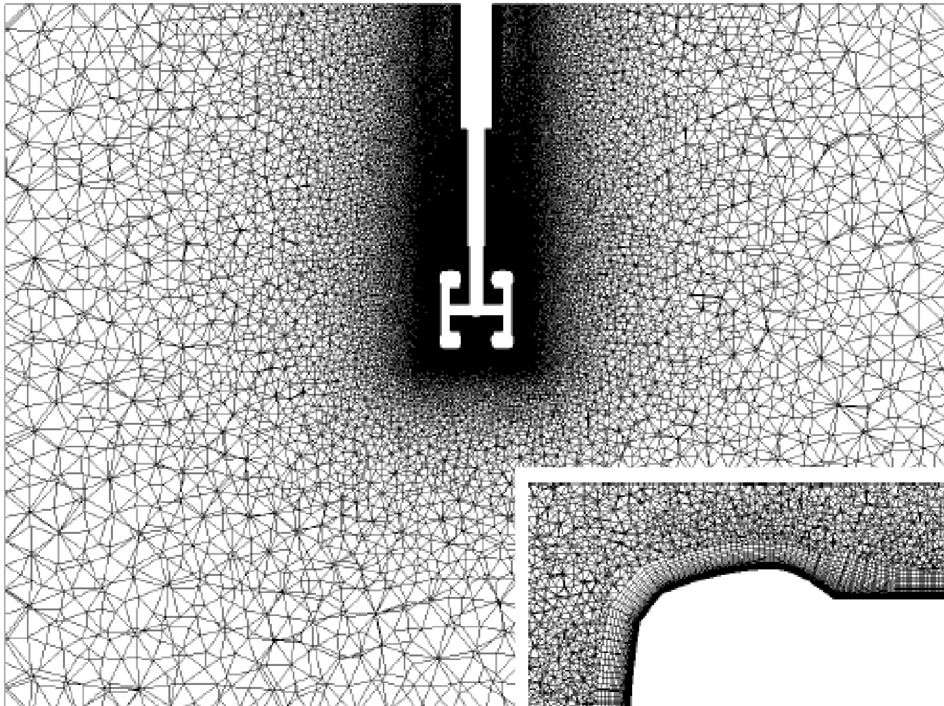


Landing Gear Noise Prediction

Landing Gear Noise Prediction

- Detached eddy simulation (DES): BCFD code
  - hybrid unstructured mesh (finite volume)
  - second order spatial discretization using HLLE plus bounded central difference
  - Barth-Jespersen flux limiter
  - dual time stepping with implicit line Gauss-Seidel scheme for pseudo time and optimized mixed 2^{nd} - 3^{rd} order backward-differencing formulation for physical time step
  - SA turbulence model on RANS regions

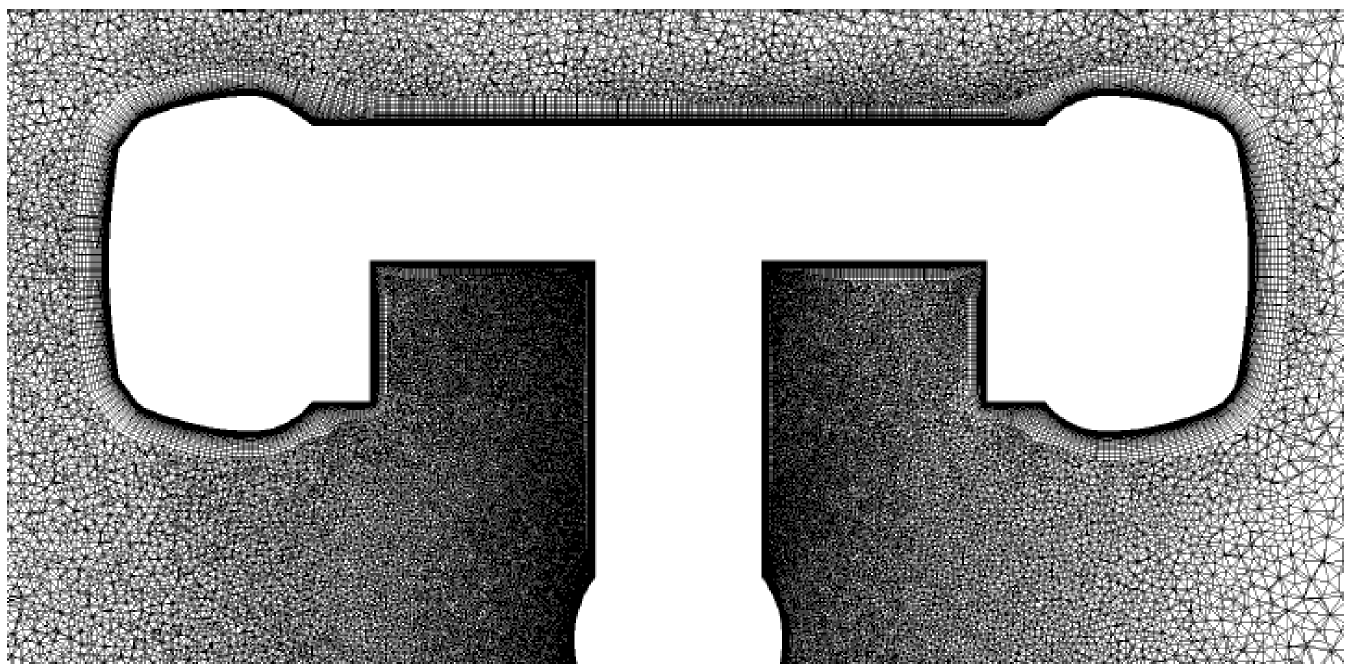
Landing Gear Noise Prediction



Mesh with approx. 100 million volumes

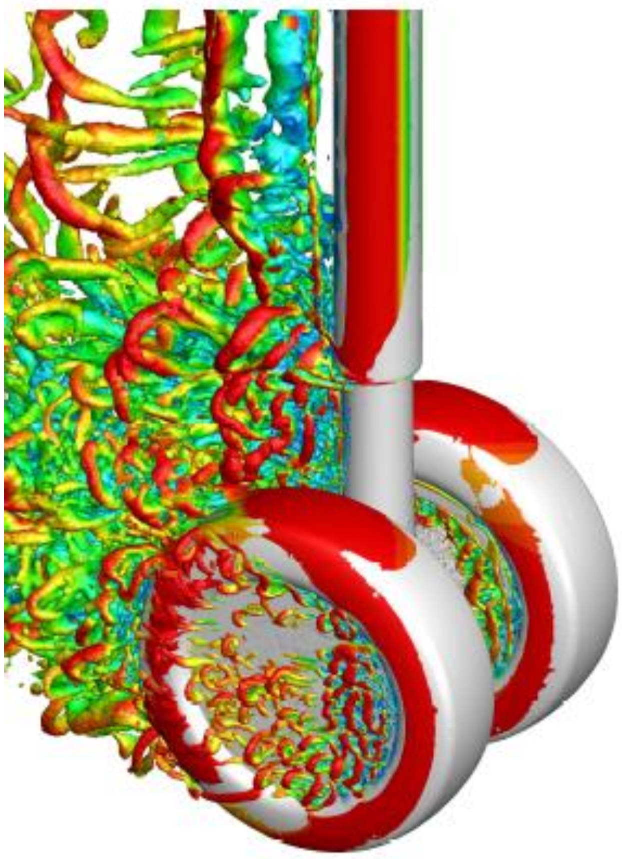
Boundary layers: prism elements

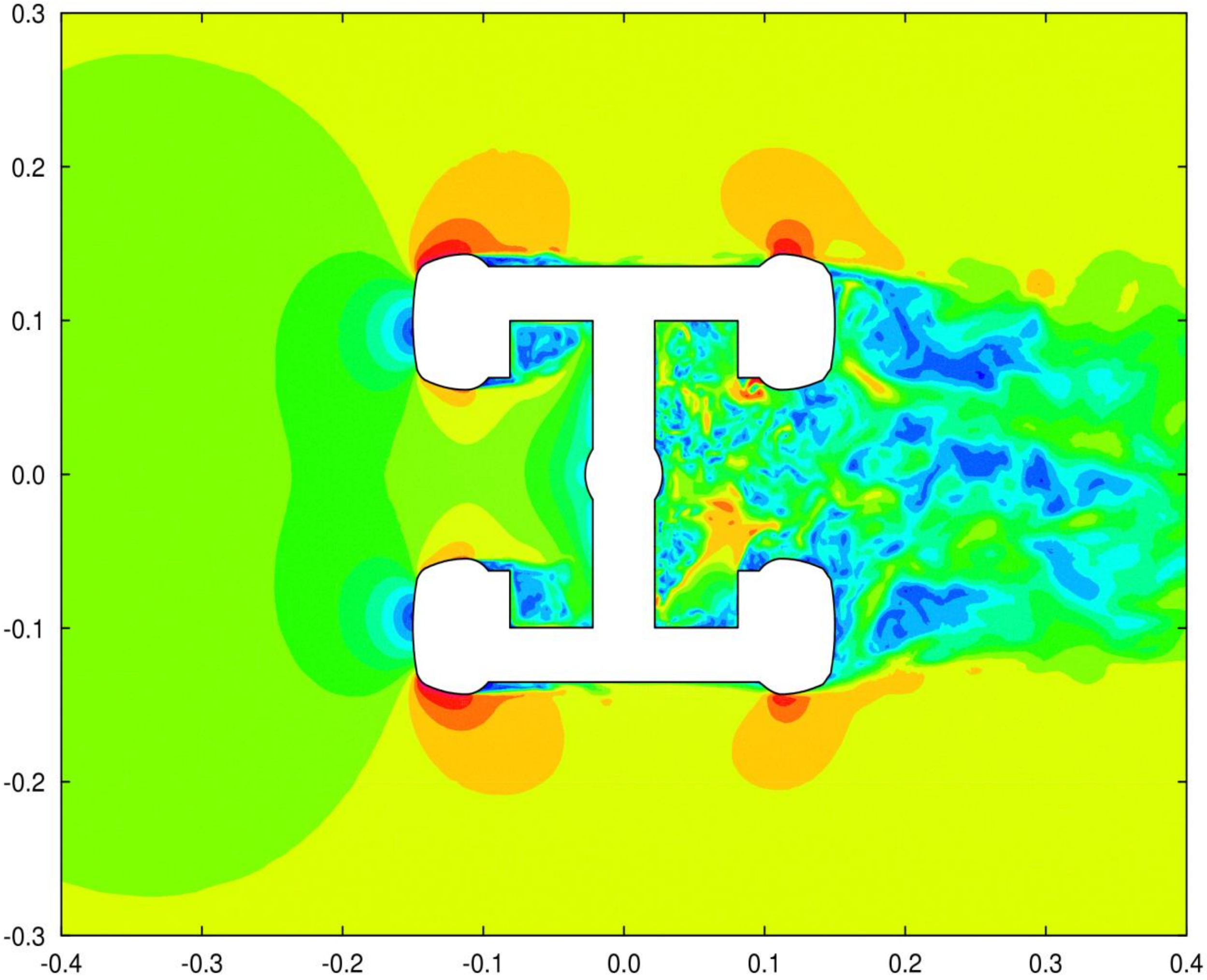
Wakes: tetrahedral elements



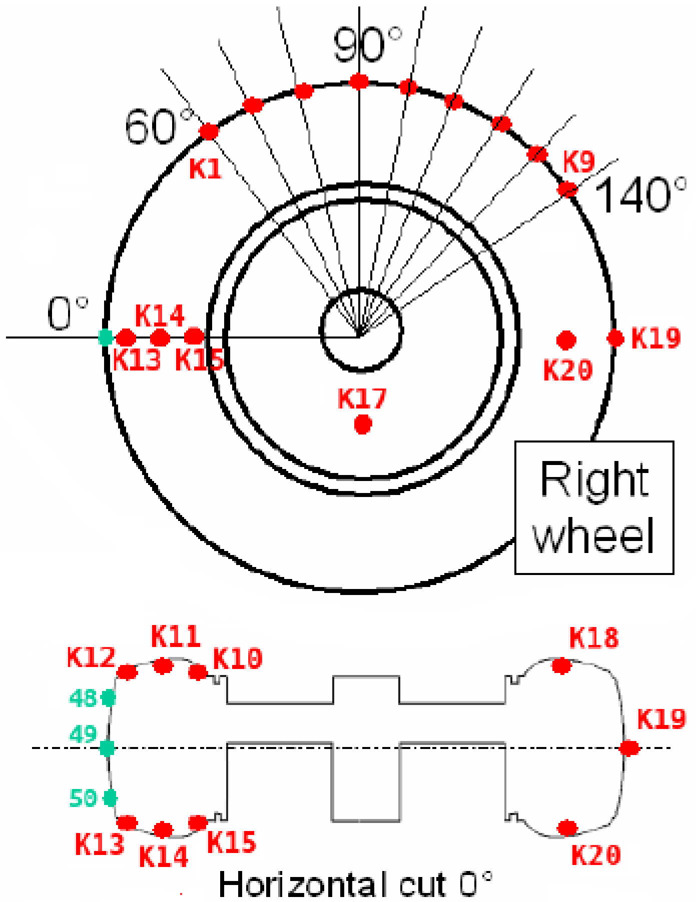
Landing Gear Noise Prediction

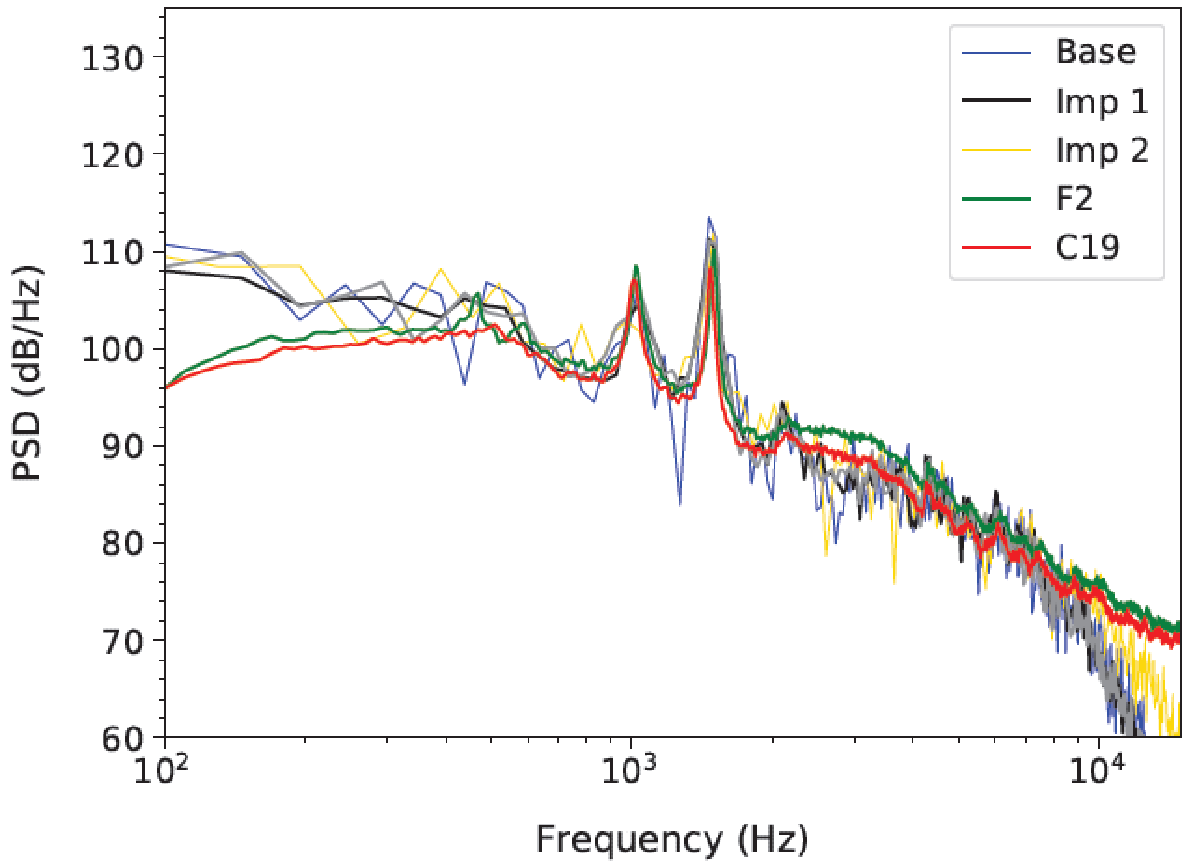
LAGOON landing gear DES by Ricciardi, *et al*. (2017)





Landing Gear Noise Prediction

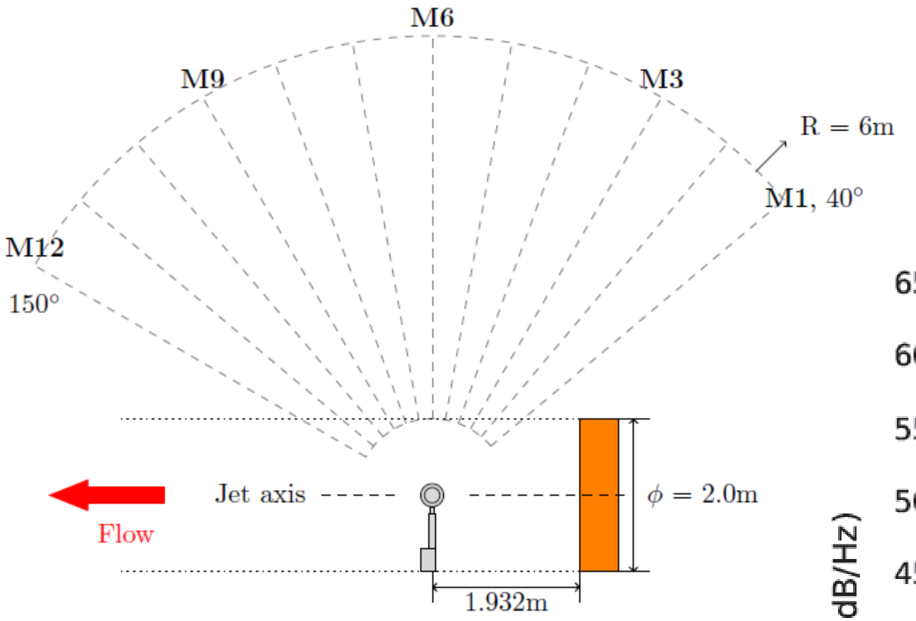




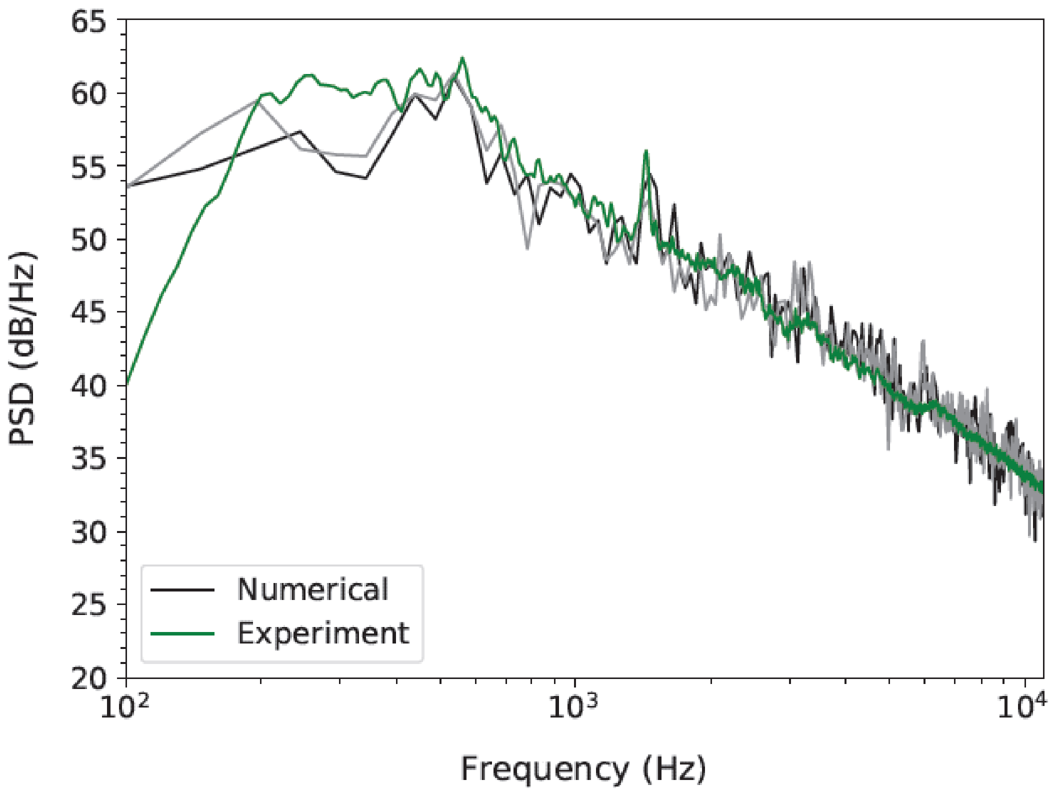
Adapted from Manoha et al., 2009

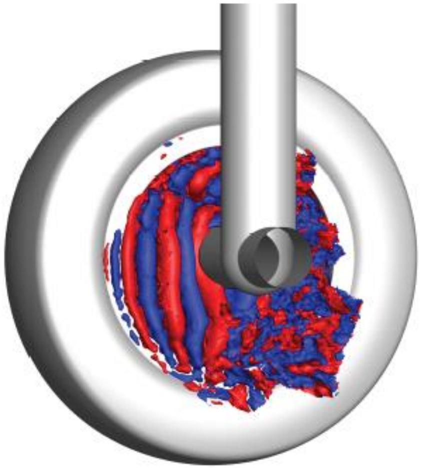
Near-field surface pressure (K15 probe)

Landing Gear Noise Prediction



Far-field pressure
M6 microphone - sideline





Coherent structures from SPOD at 1500 Hz

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Ffowcs Williams & Hall Analogy

- Lighthill showed that free turbulence radiates noise with $\bar{p}'^2 \sim M^8$ while Curle showed that a compact surface radiates noise with $\bar{p}'^2 \sim M^6$
- Ffowcs Williams & Hall investigated the case of turbulence in the proximity of a trailing edge

$$\nabla^2 \widehat{p'} + k^2 \widehat{p'} = -\left[\widehat{\frac{\partial^2 \rho u_i u_j}{\partial x_i \partial x_j}} \right] \qquad k = \frac{\omega}{c_0}$$

Non-homogeneous Helmholtz equation; quiescent medium

Ffowcs Williams & Hall Analogy

- Acoustic scattering problem needs to be solved

$$\nabla^2 G + k^2 G = \delta(\mathbf{x} - \mathbf{y}) \longrightarrow$$

Green's function: fundamental solution of Helmholtz eqn.

Making use of Green's second theorem one can write

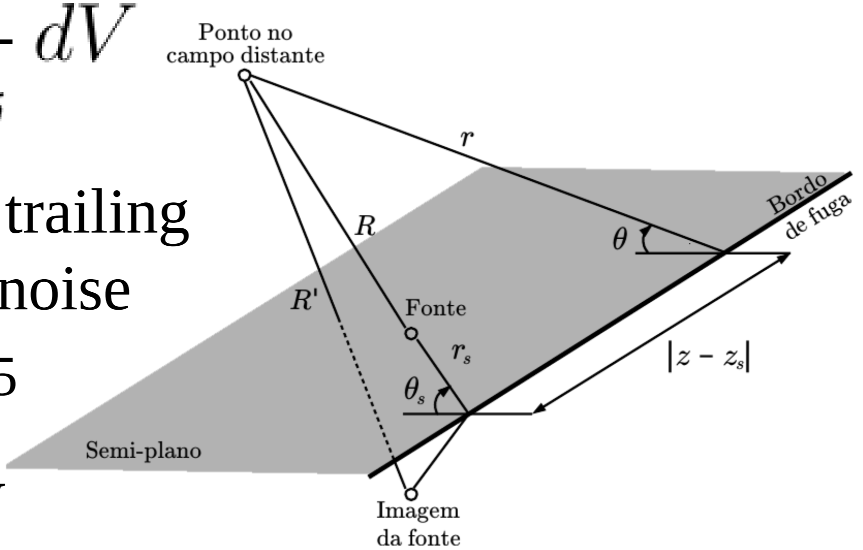
$$\int_V G \nabla^2 \widehat{p'} - \widehat{p'} \nabla^2 G \, dV = \oint_S G \frac{\partial \widehat{p'}}{\partial \mathbf{n}} - \widehat{p'} \frac{\partial G}{\partial \mathbf{n}} \, dS$$

$$\widehat{p'}(\mathbf{x}, \omega) = \underbrace{\oint_S G \frac{\partial \widehat{p'}}{\partial \mathbf{n}} - \widehat{p'} \frac{\partial G}{\partial \mathbf{n}} \, dS}_{\text{scattered sound}} + \underbrace{\int_V G \frac{\partial^2 \widehat{\rho u_i u_j}}{\partial x_i \partial x_j} \, dV}_{\text{incident sound}}$$

Ffowcs Williams & Hall Analogy

- If we look for the Green's function that gives
  $\partial G/\partial \mathbf{n} = 0$
- We know that for a rigid surface $\partial \widehat{p'}/\partial \mathbf{n} = 0$
- Ffowcs Williams & Hall equation provides

$$\widehat{p'}(\mathbf{x}, \omega) = \int_V \widehat{\rho u_i u_j} \frac{\partial^2 G}{\partial x_i \partial x_j} dV$$



Turbulent eddy in the proximity of a trailing edge is a good model for TBL airfoil noise

- important result: $\overline{p}'^2 \sim M^5$
- cardioid shape directivity

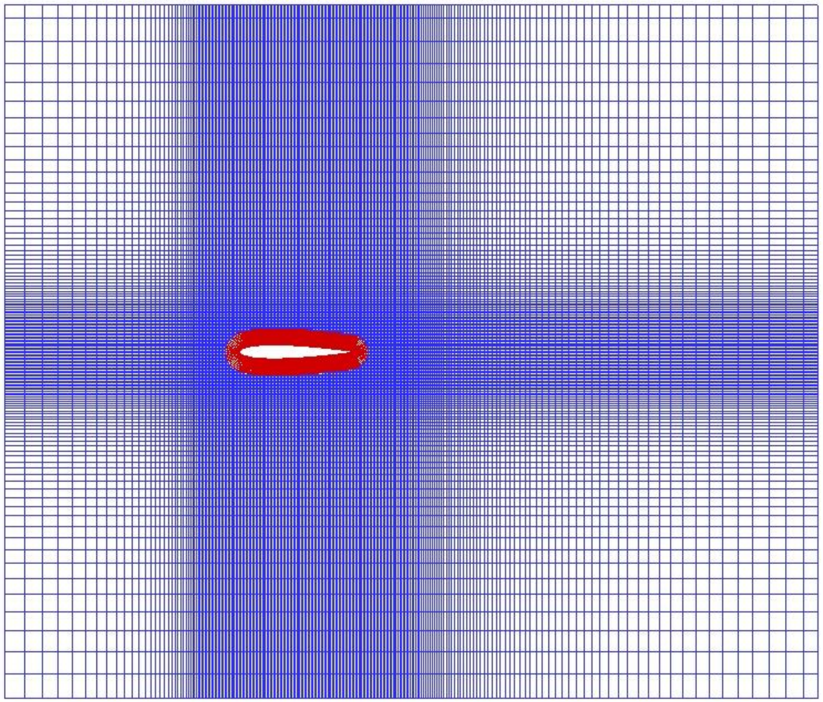
Airfoil Noise Prediction

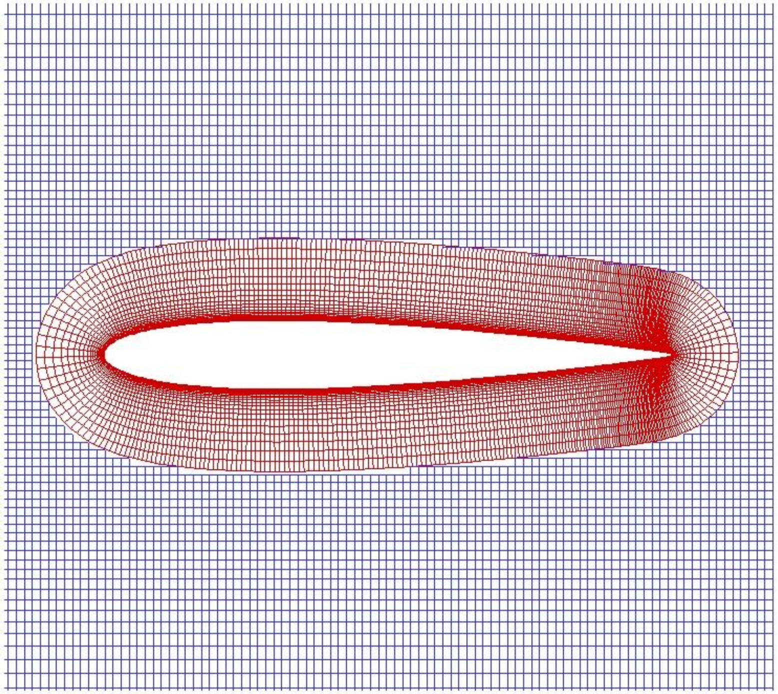
Airfoil Noise Prediction

- **General curvilinear form of compressible Navier-Stokes equations**
- **Spatial discretization:** 6th order compact scheme on staggered grid plus 6th order compact schemes for filtering and interpolation
- **Overset grid:** 4th order Hermitian interpolation
- **Time integration:** Near-wall region – implicit 2nd order Beam-Warming scheme; Away from solid boundaries – 3rd order Runge-Kutta scheme
- **Unresolved turbulent scales:** Dynamic SGS model
- **Noise prediction:** Ffowcs Williams and Hawkings acoustic analogy

Airfoil Noise Prediction

$Re_c = 408000$, $Ma_\infty = 0.115$, AOA = 0 deg.





Every 4-th grid point

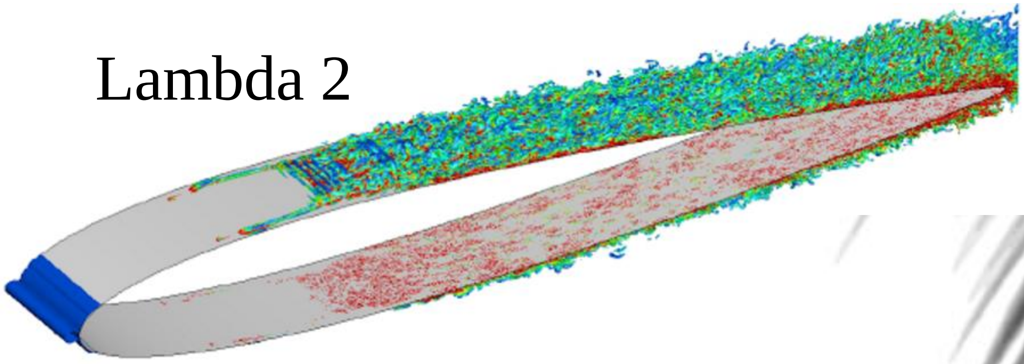
O-grid block – 1536 x 125 x 128

Background grid block – 896 x 511 x 64

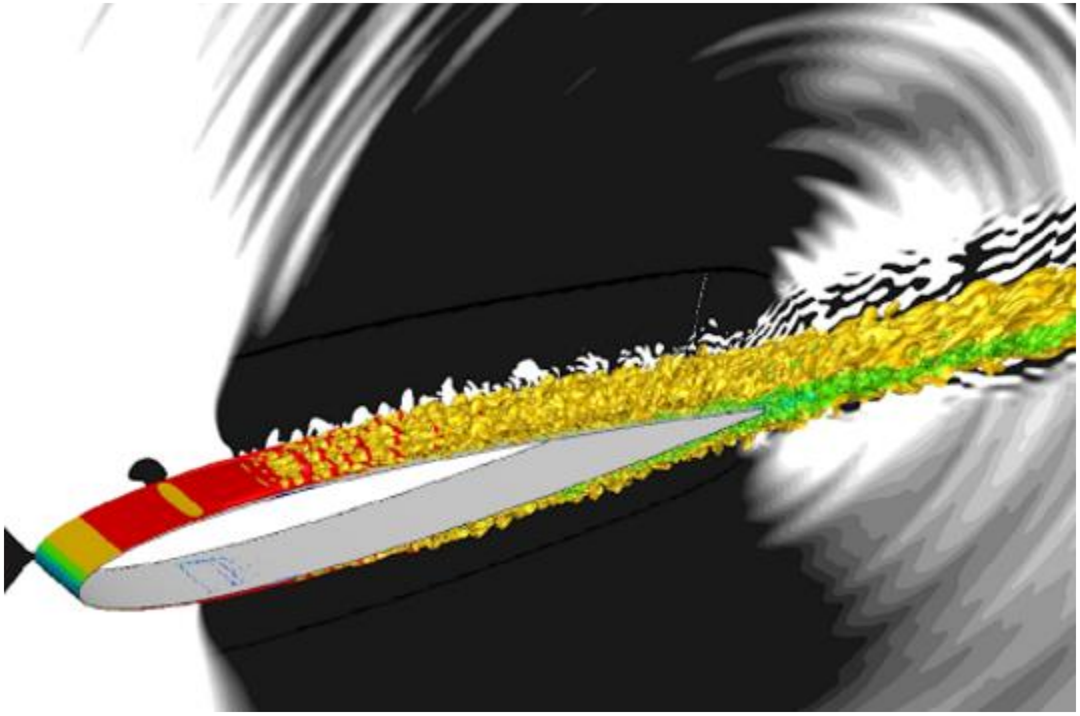
Airfoil Noise Prediction

$Re_c = 408000$, $Ma_\infty = 0.115$, AOA = 0 deg.

Lambda 2

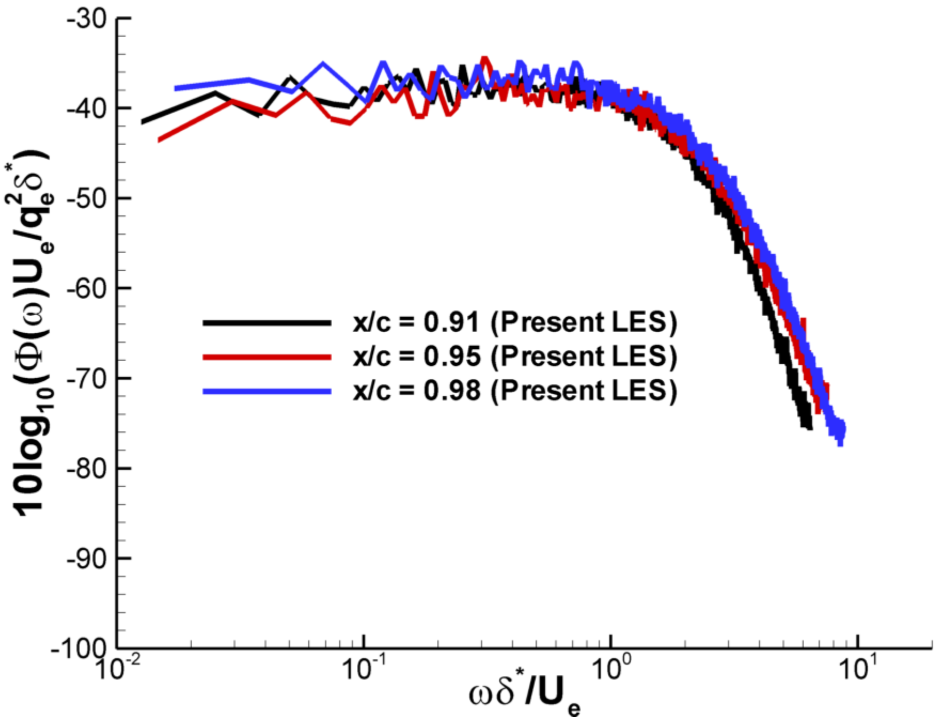


Iso-surfaces of vorticity plus dilatation field

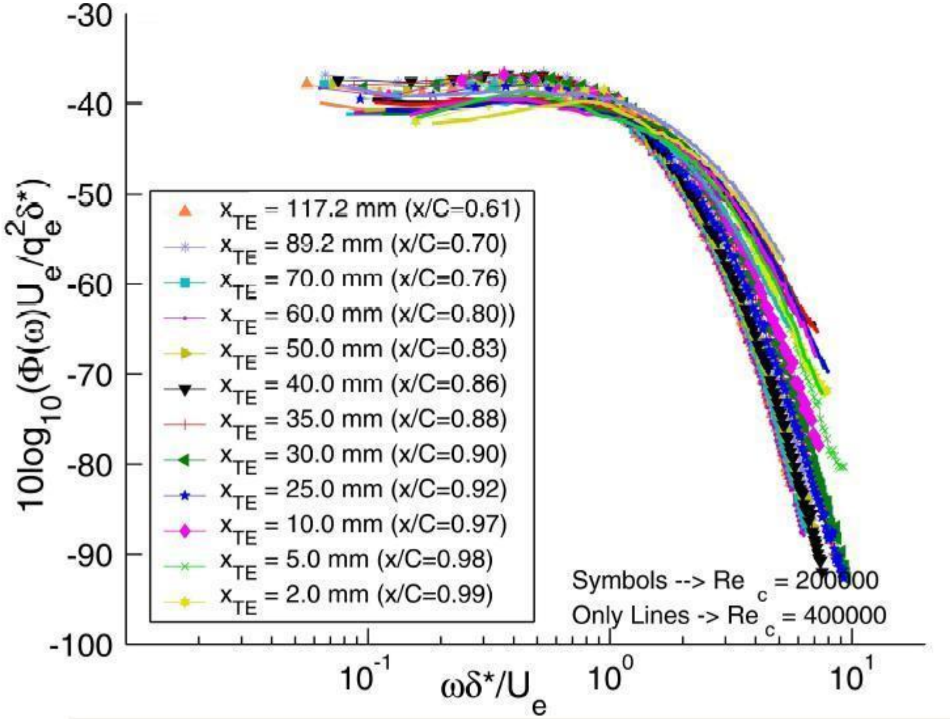


Airfoil Noise Prediction

Wall pressure PSD normalized by outer variables



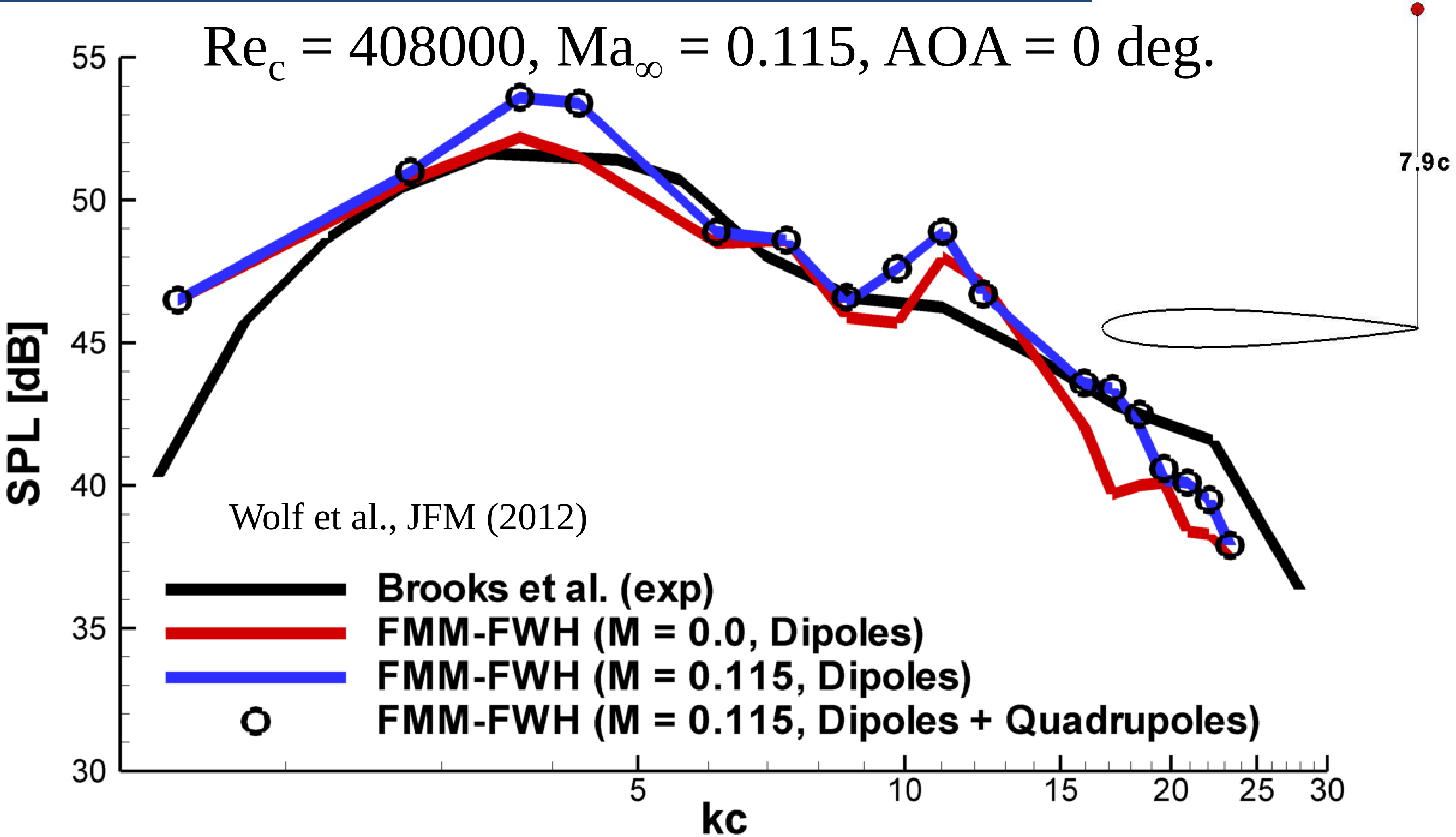
Present LES



Experiments from Sagrado and Hynes[1]

[1] Sagrado and Hynes, Fluids and Structures, Vol. 30, 2012, pp. 3-34

Airfoil Noise Prediction

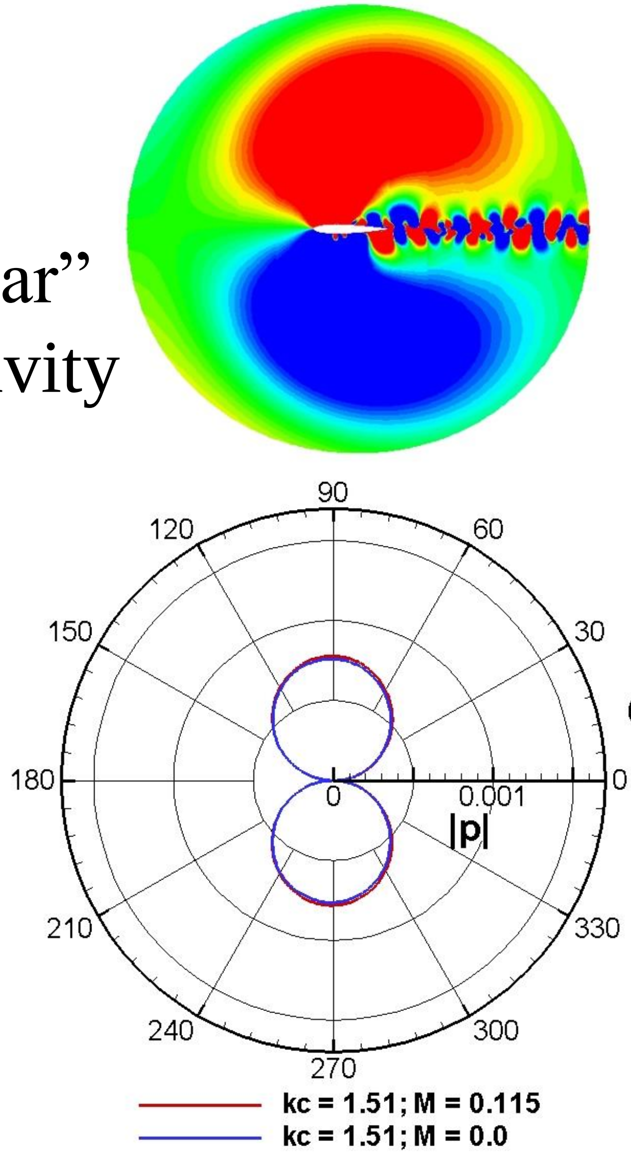


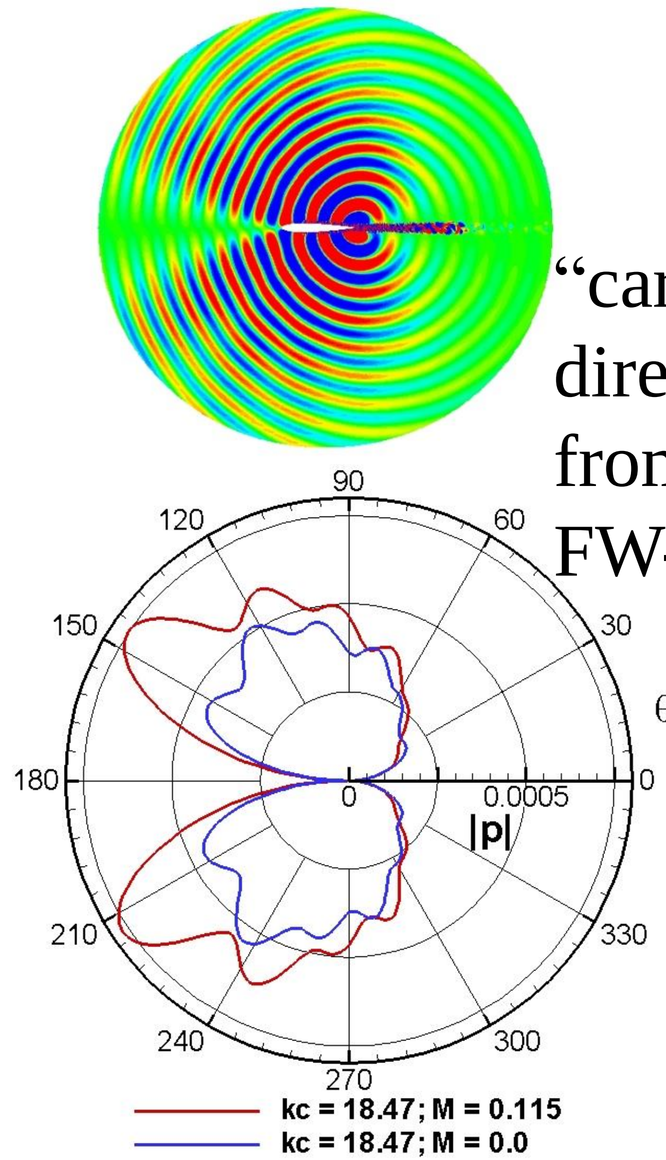
Sound pressure level at x = c, y = 7.9c and mid-span

[1] Brooks *et al.*, NASA Publication, 1989

Airfoil Noise Prediction

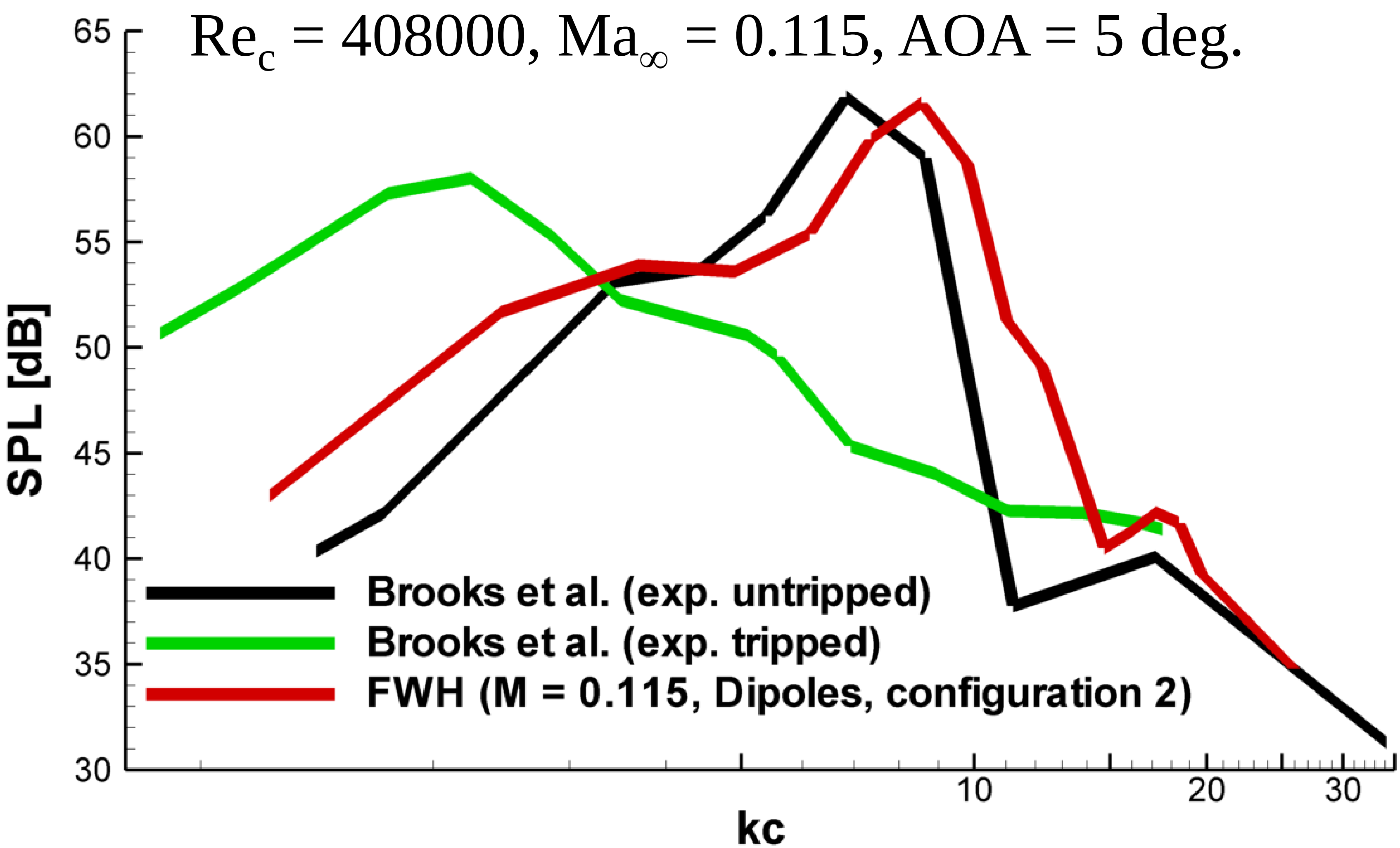
“dipolar” directivity from Curle





“cardioid” directivity from FW-Hall

Airfoil Noise Prediction



Sound pressure level at x = c, y = 7.9c and mid-span

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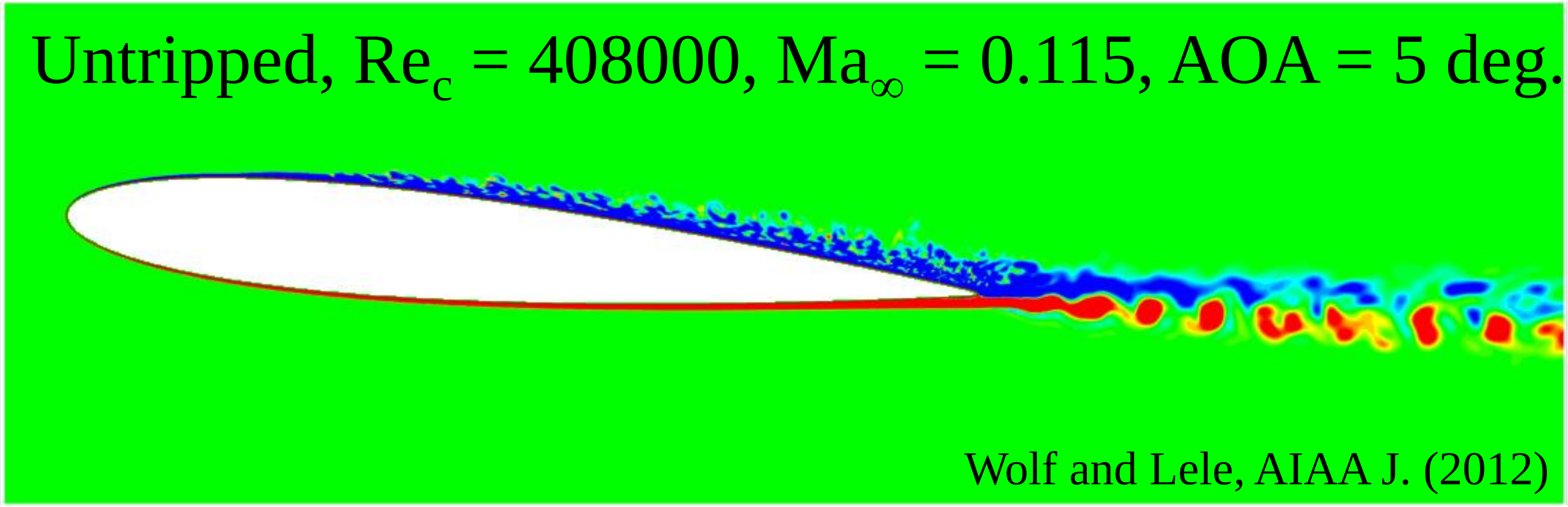
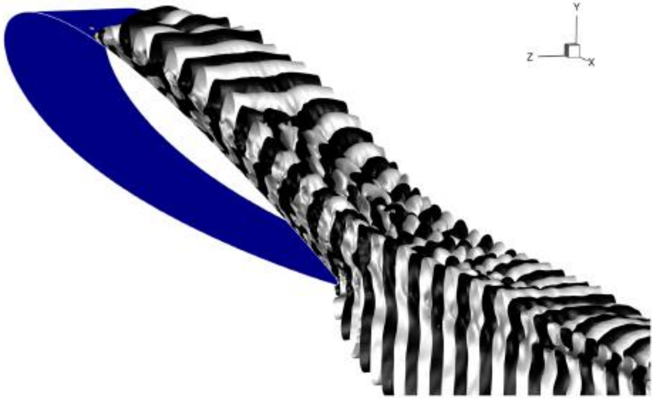
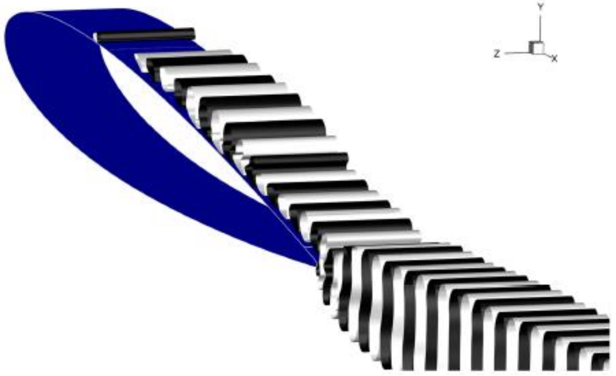
Airfoil Noise Prediction



1st POD mode, TKE norm



1st SPOD mode, TKE norm



Ribeiro and Wolf, PoF (2017)

Poro-Elastic Trailing Edges

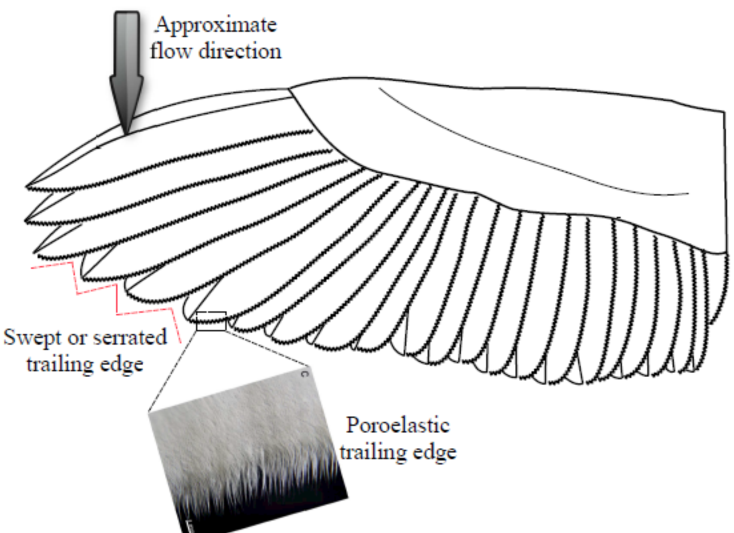
Pigeon

Owl

BBC TWO – Natural World 2015-2016, Super Powered Owls - Flying silently

Poro-Elastic Trailing Edges

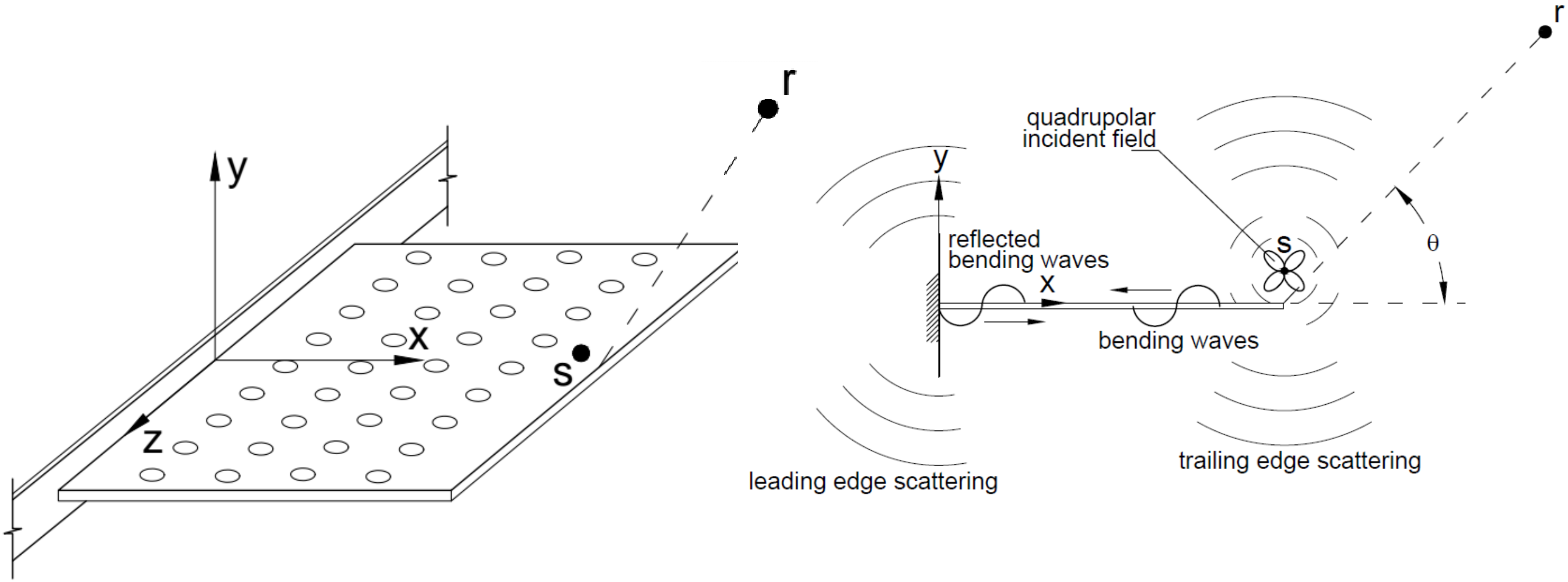
Owls are silent hunters



- Owls possess the ability to fly and hunt silently
- Their low self-noise is due to serrated trailing-edge and poro-elastic feather structure
- Several authors studied the reduction in noise scattering by application of porosity and elasticity assuming semi-infinite extent of elastic or poro-elastic edge

Poro-Elastic Trailing Edges

Problem description



Wolf and Cavalieri, AIAA P (2015)

Poro-Elastic Trailing Edges

- Three equations coupled:
  - Vibration of plate subject to acoustic load
  - Acoustic scattering due to noise source
  - Euler equation for coupling between fluid and structure

$$(1-\alpha_H)\nabla^4\eta - \frac{k_0^4}{\Omega^4}\eta = (1+\alpha_H K_R)\frac{\epsilon}{\Omega^6}k_0^3\Delta p$$

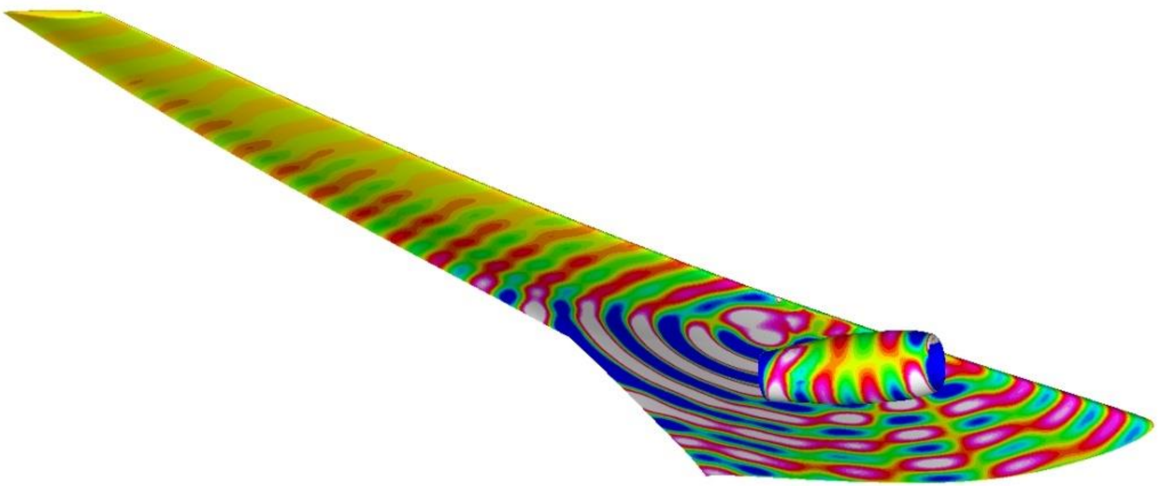
$$\Omega = (\tilde{\omega}/\tilde{\omega_c})^{1/2} = \tilde{k}_0/\tilde{k}_B \qquad \nabla^2 p + k_0^2 p = -S$$

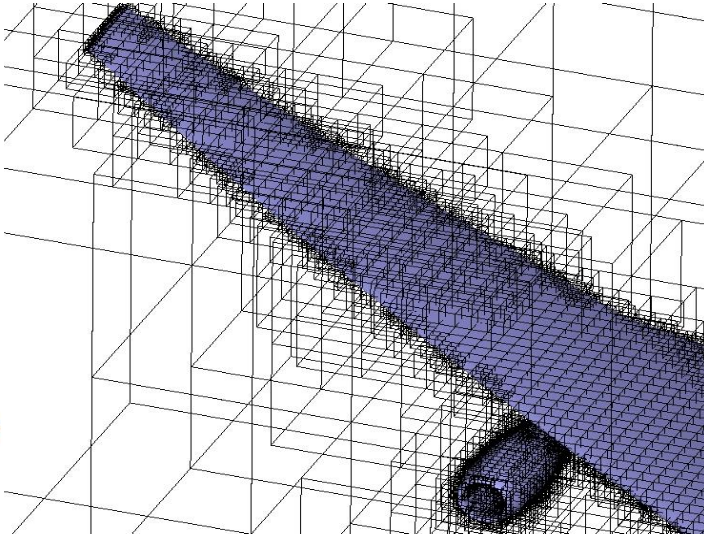
$$\epsilon = \frac{\tilde{\rho}_f \tilde{k}_0}{\tilde{m}\tilde{k}_B^2} \qquad (1-\alpha_H)k_0^2\eta - \frac{\alpha_H K_R}{2R}\Delta p = \left.\frac{\partial p}{\partial y}\right|_{y=0}$$

Poro-Elastic Trailing Edges

Numerical methodology for 3D problem

- Vibration problem solved *a priori*: structural modal basis obtained by finite-element method
- Acoustic problem solved using boundary element method accelerated by fast multipole method
- Euler equation provides boundary condition for BEM





Poro-Elastic Trailing Edges

Helmholtz integral equation

$$T(\mathbf{x})p(\mathbf{x}) = \int_{\Gamma} \left[\frac{\partial p(\mathbf{y})}{\partial n_y} G(\mathbf{x},\mathbf{y}) - \frac{\partial G(\mathbf{x},\mathbf{y})}{\partial n_y} p(\mathbf{y}) \right] \mathrm{d}\Gamma - \frac{\partial^2 G(\mathbf{x},\mathbf{z}_i)}{\partial \mathbf{z}_{i_m} \partial \mathbf{z}_{i_n}} S(\mathbf{z}_i)$$

Boundary condition

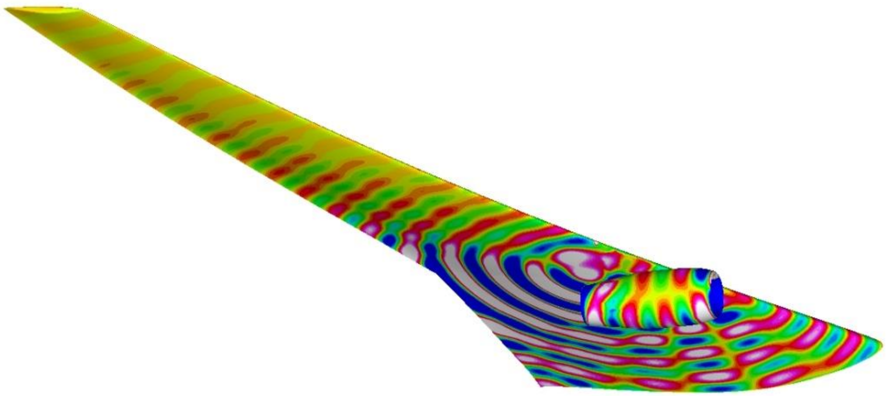
$$\left.\frac{\partial p}{\partial n}\right|_{\Gamma} = n_y \left.\frac{\partial p}{\partial y}\right|_{y=0} = n_y (1+\alpha_{\mathrm{H}} K_{\mathrm{R}}) \frac{\epsilon k_0^5}{\Omega^6} \frac{\langle \Delta p(x), \phi_j \rangle}{\beta_j^4 - k_0^4/(1-\alpha_{\mathrm{H}})\Omega^4} \phi_j - n_y \frac{\alpha_{\mathrm{H}} K_{\mathrm{R}}}{2R} \Delta p$$

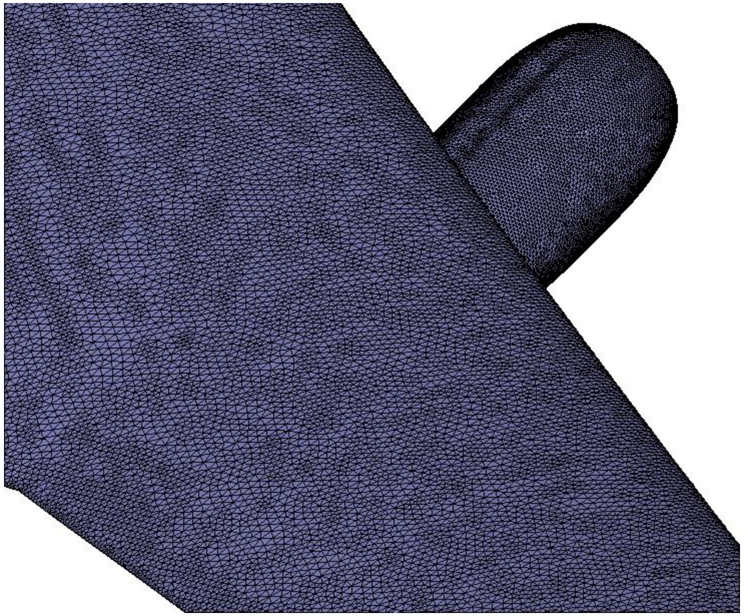
BEM Linear system

$$[H]\{p\} - [G]\left\{\frac{\partial p}{\partial n}\right\} = \{S\} \longrightarrow ([H]-[G][D])\{p\} = \{S\}$$

$$D_{i,k} = (1+\alpha_{\mathrm{H}} K_{\mathrm{R}}) \frac{\epsilon k_0^5}{\Omega^6} n_{y_i} n_{y_k} \gamma_k \sum_{j=1}^{M} \frac{\phi(x_k)_j \phi(x_i)_j}{\beta_j^4 - k_0^4/(1-\alpha_{\mathrm{H}})\Omega^4} - \frac{\alpha_{\mathrm{H}} K_{\mathrm{R}}}{2R} n_{y_i} (n_{y_k} \delta_{k,i} + n_{y_k} \delta_{k,N-i})$$

Poro-Elastic Trailing Edges



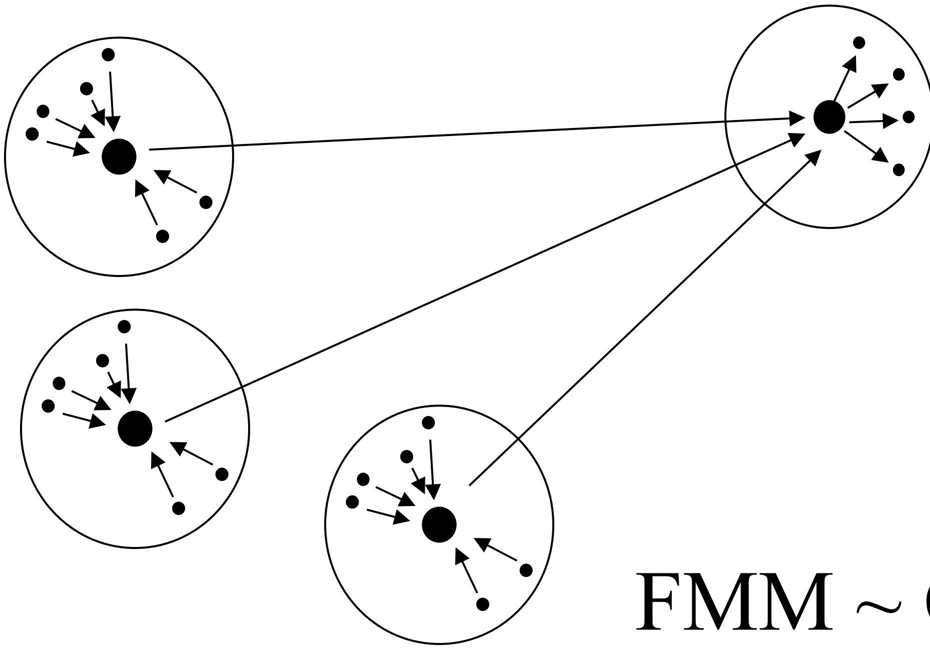


- Scattering and shielding of complex configurations
- simplified pre-processing
- accurate modeling of infinite domains
- no dispersion or dissipation of wave propagation
- large scale problems are expensive - $O(N^2)$

Poro-Elastic Trailing Edges

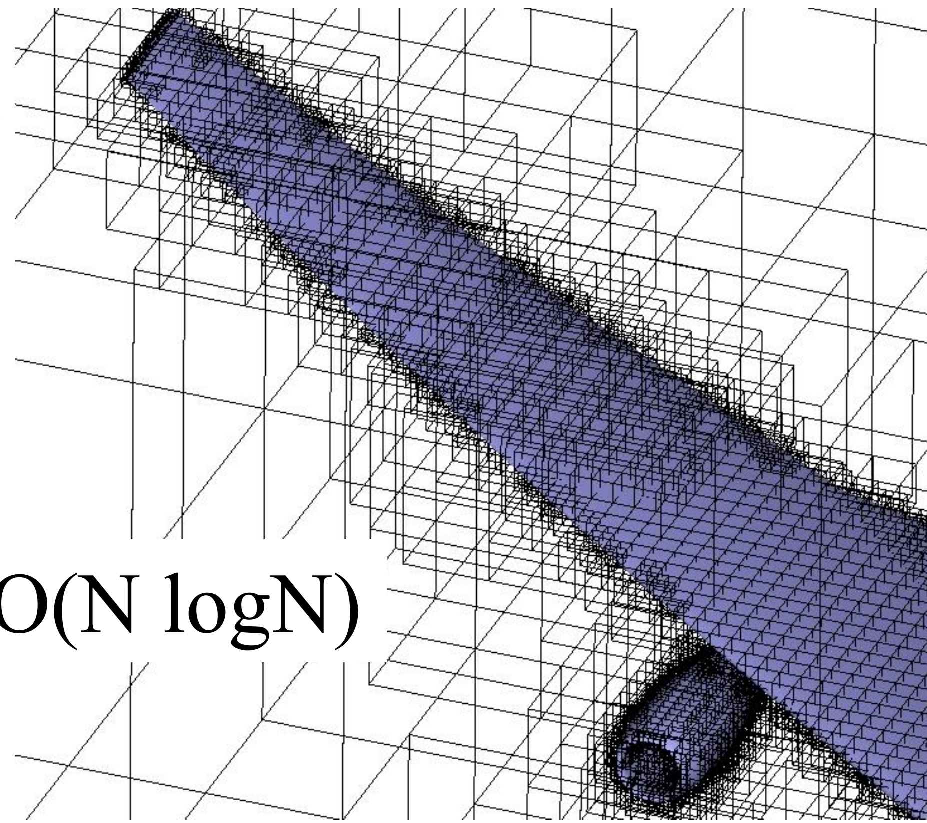
- Clustering of acoustic sources at different spatial lengths
- Effects of distant clusters evaluated at observer locations

Observer locations



Multipole expansions of acoustic sources

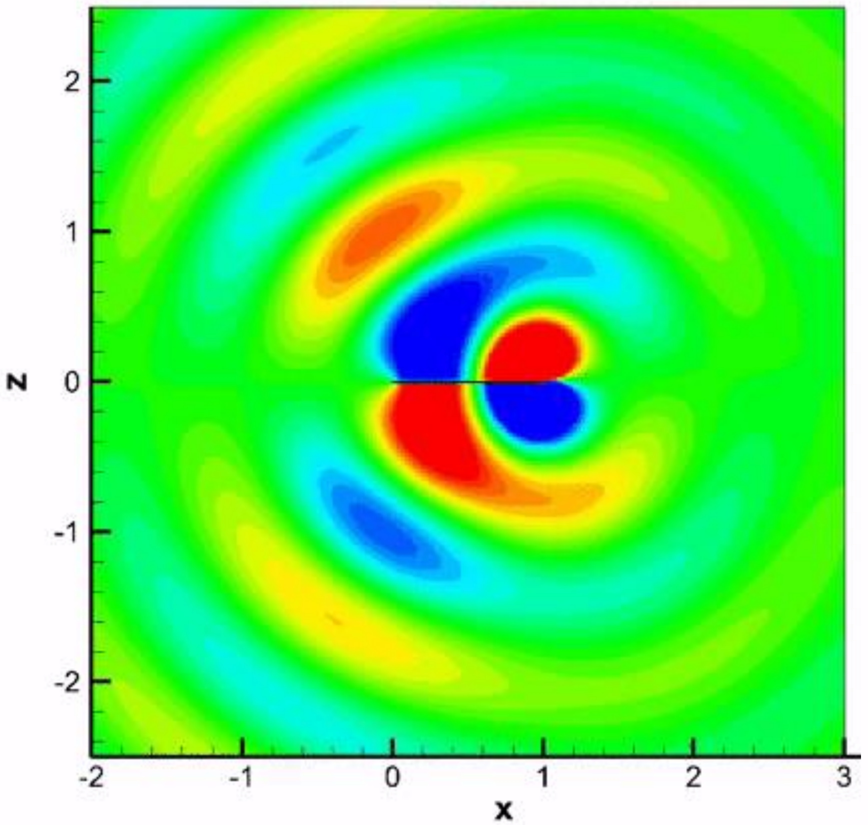
FMM ~ O(N logN)



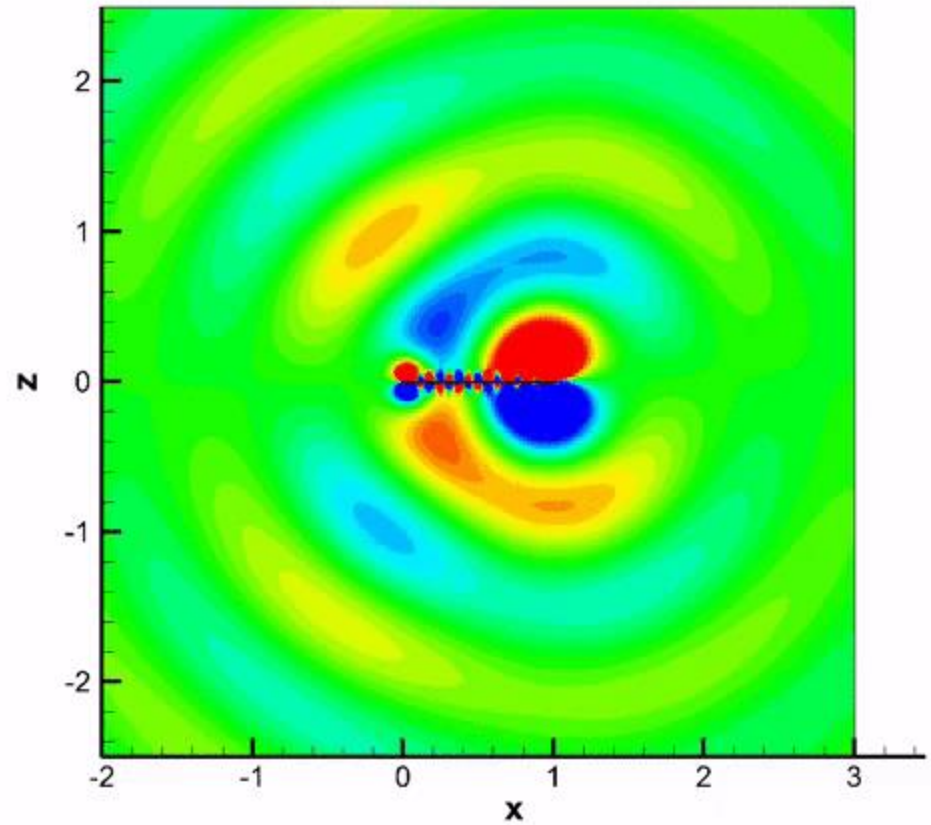
Poro-Elastic Trailing Edges

Acoustic pressure for $k_0 = 5$, AR = 1 – lateral view

3D quadrupole source placed at trailing edge



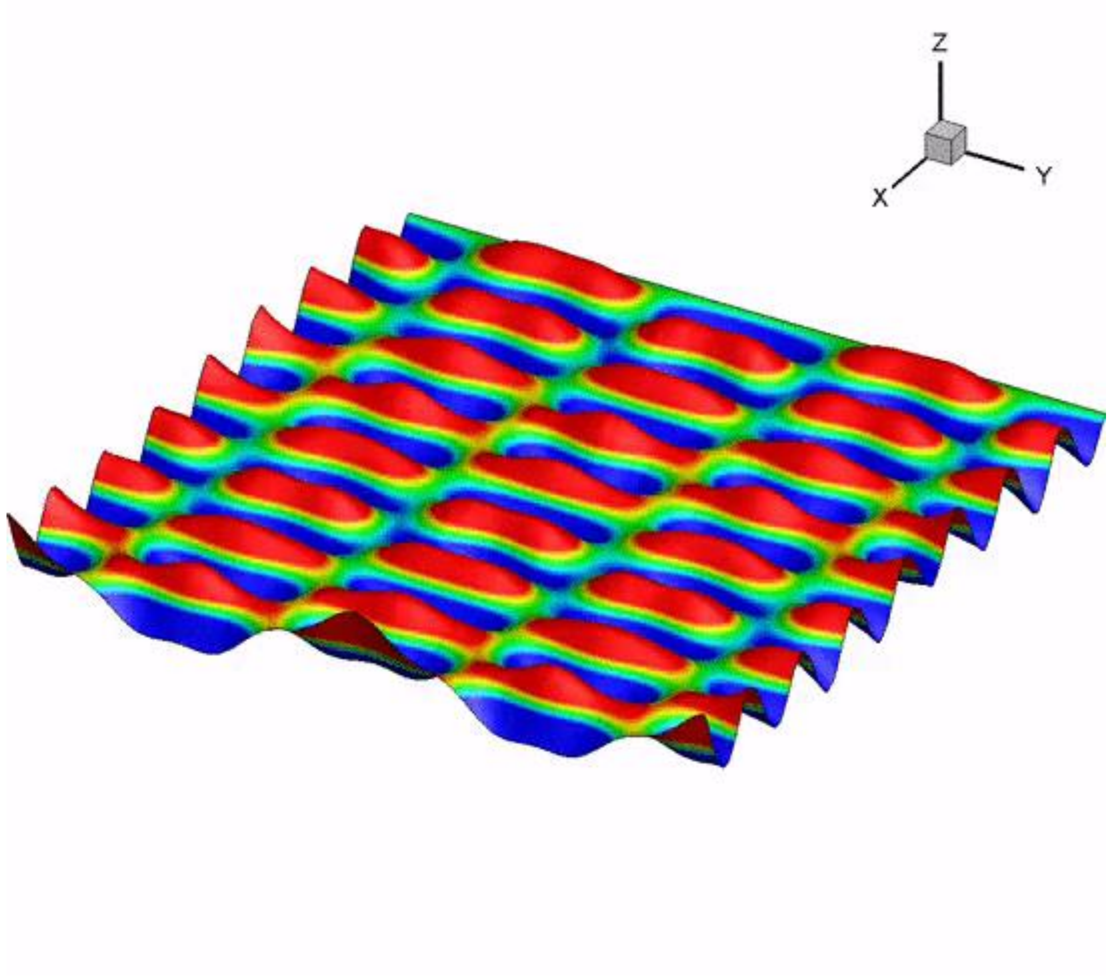
Rigid plate



Poroelastic plate $\Omega = 0.1$

Poro-Elastic Trailing Edges

Plate displacement due to 3D source



Poro-Elastic Trailing Edges

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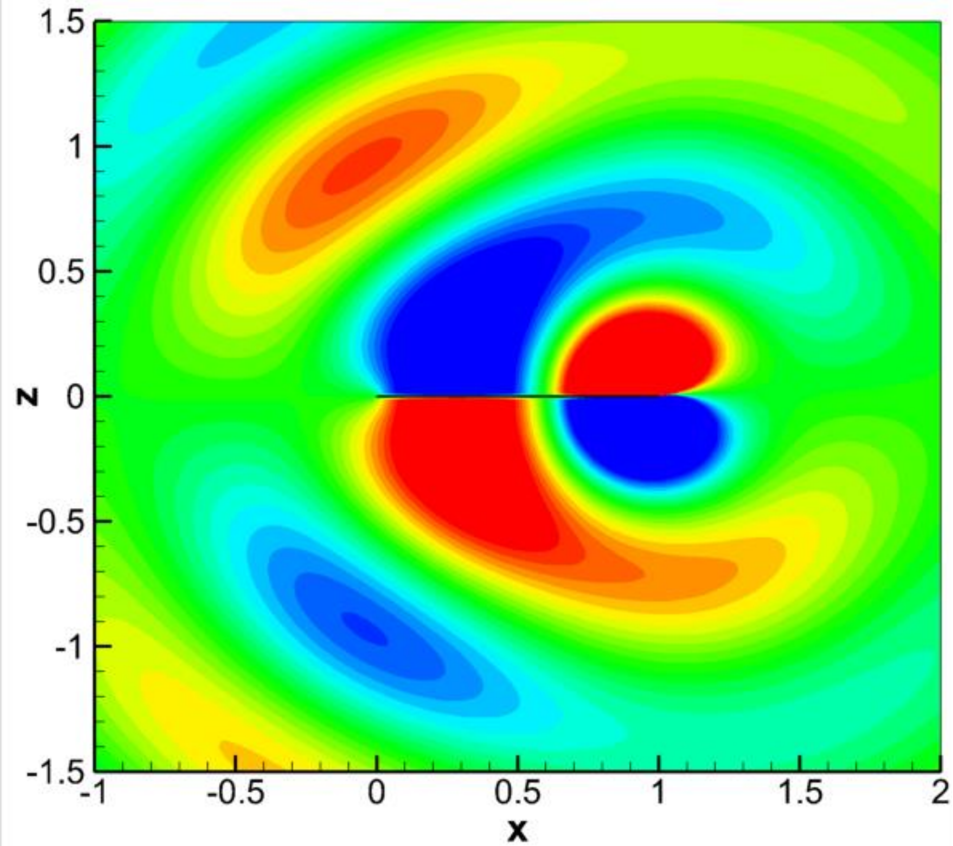
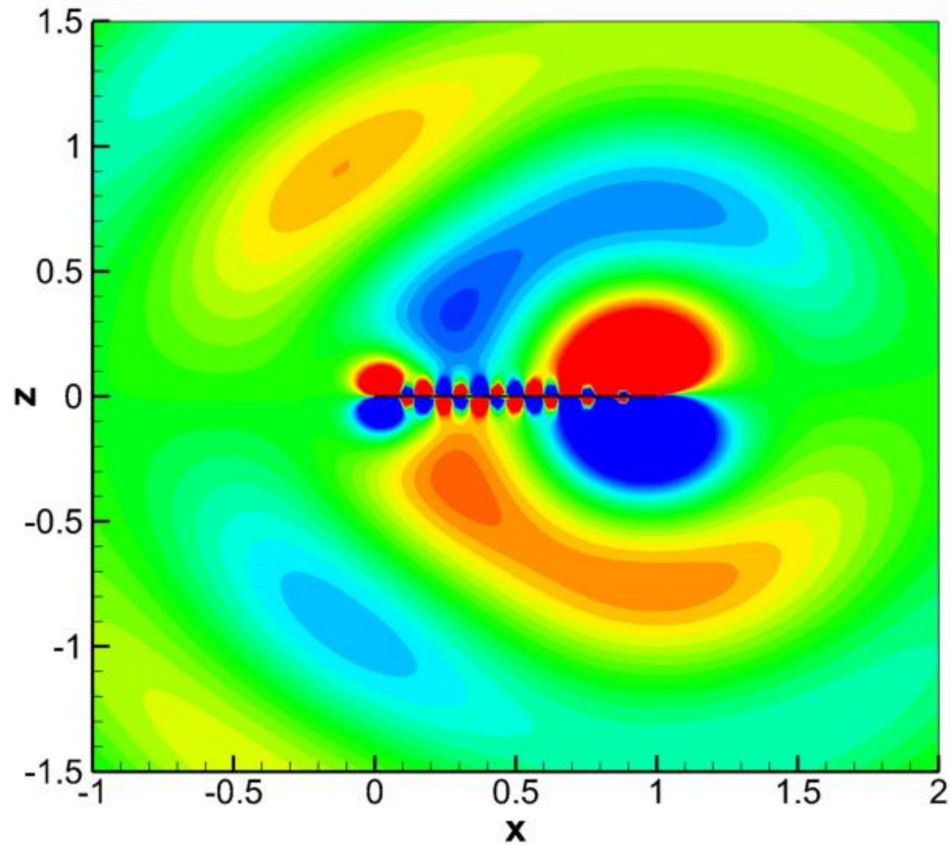


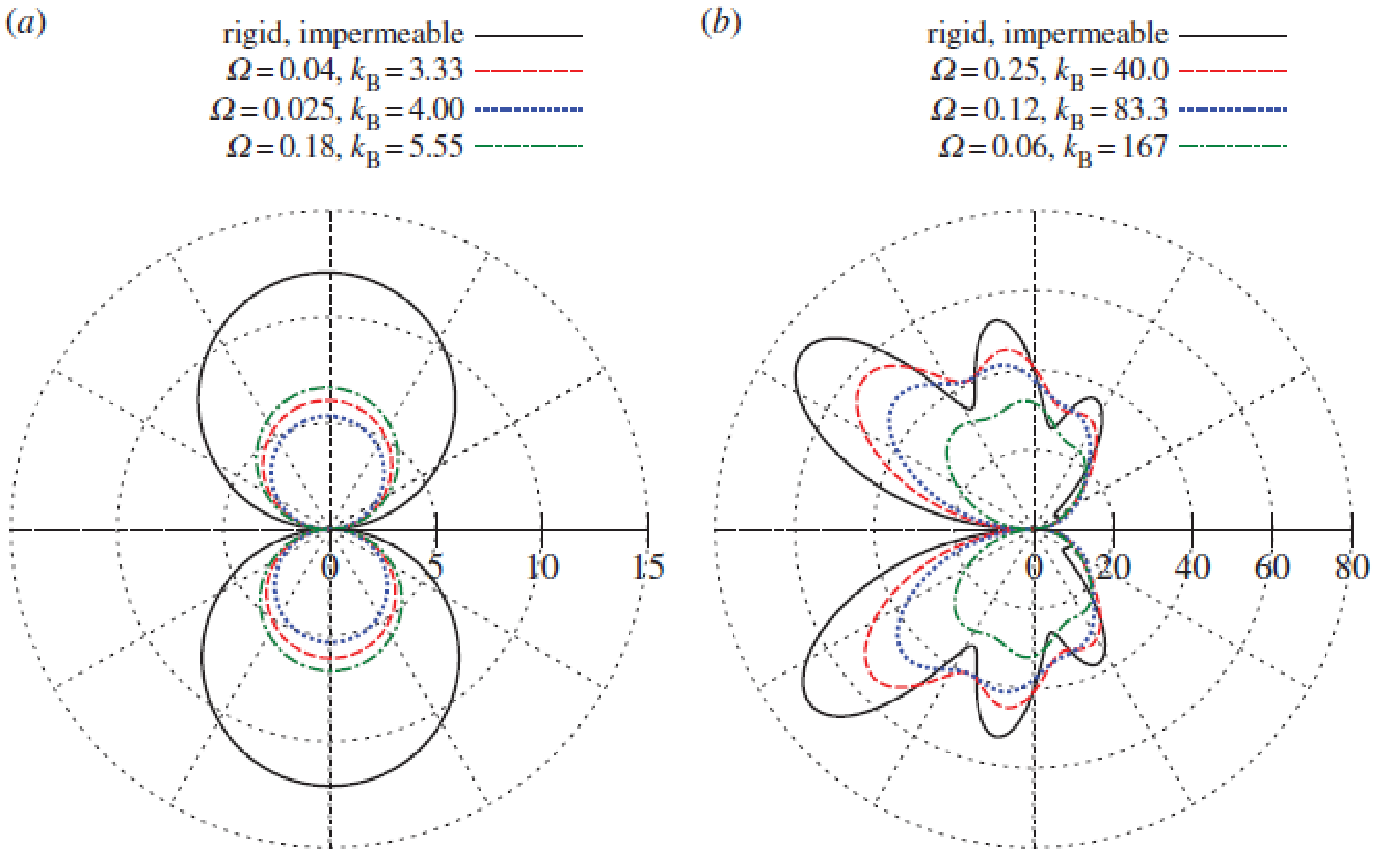
Rigid plate



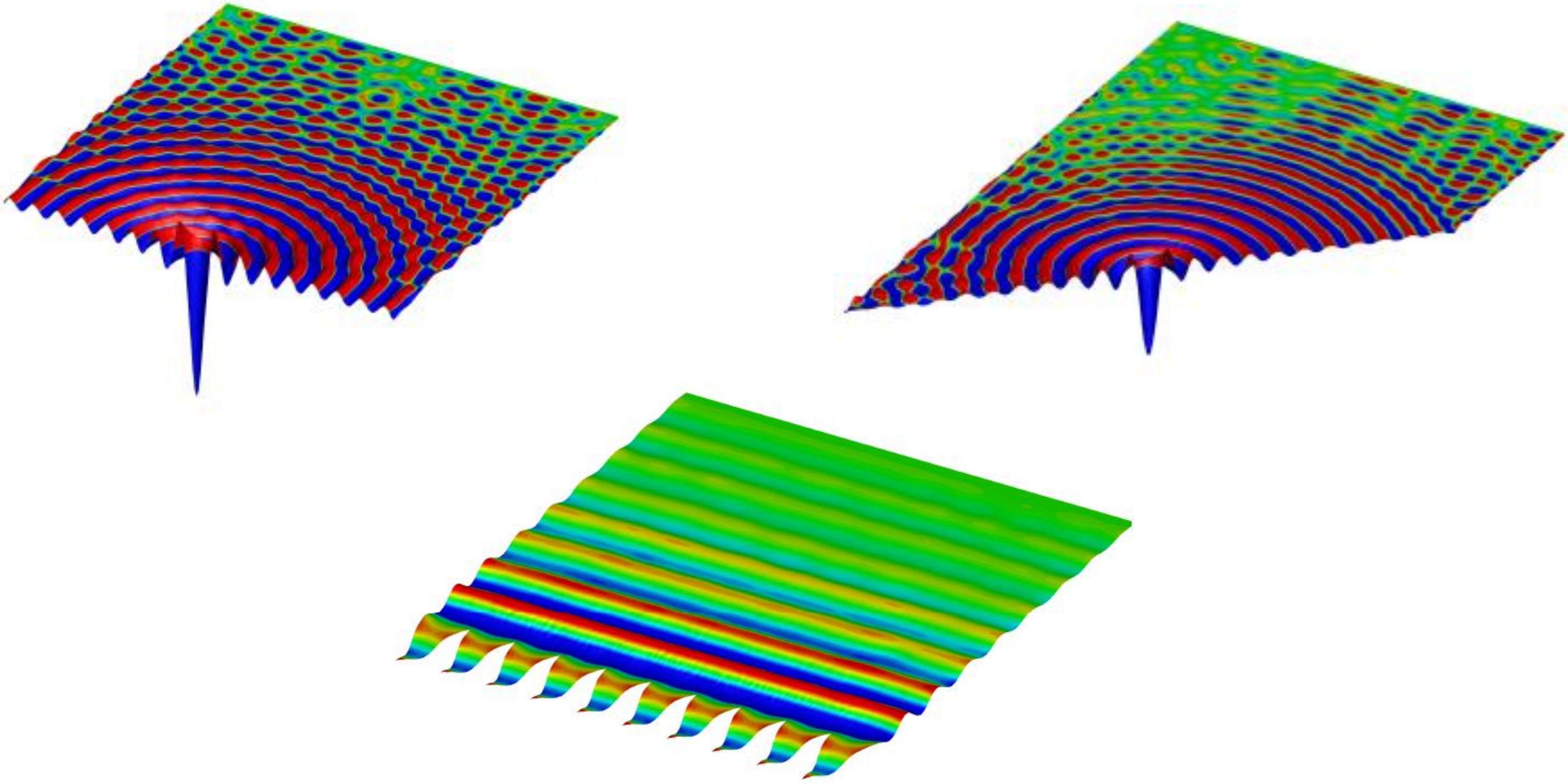
Poroelastic plate $\Omega = 0.1$

Poro-Elastic Trailing Edges

Finite poro-elastic plates

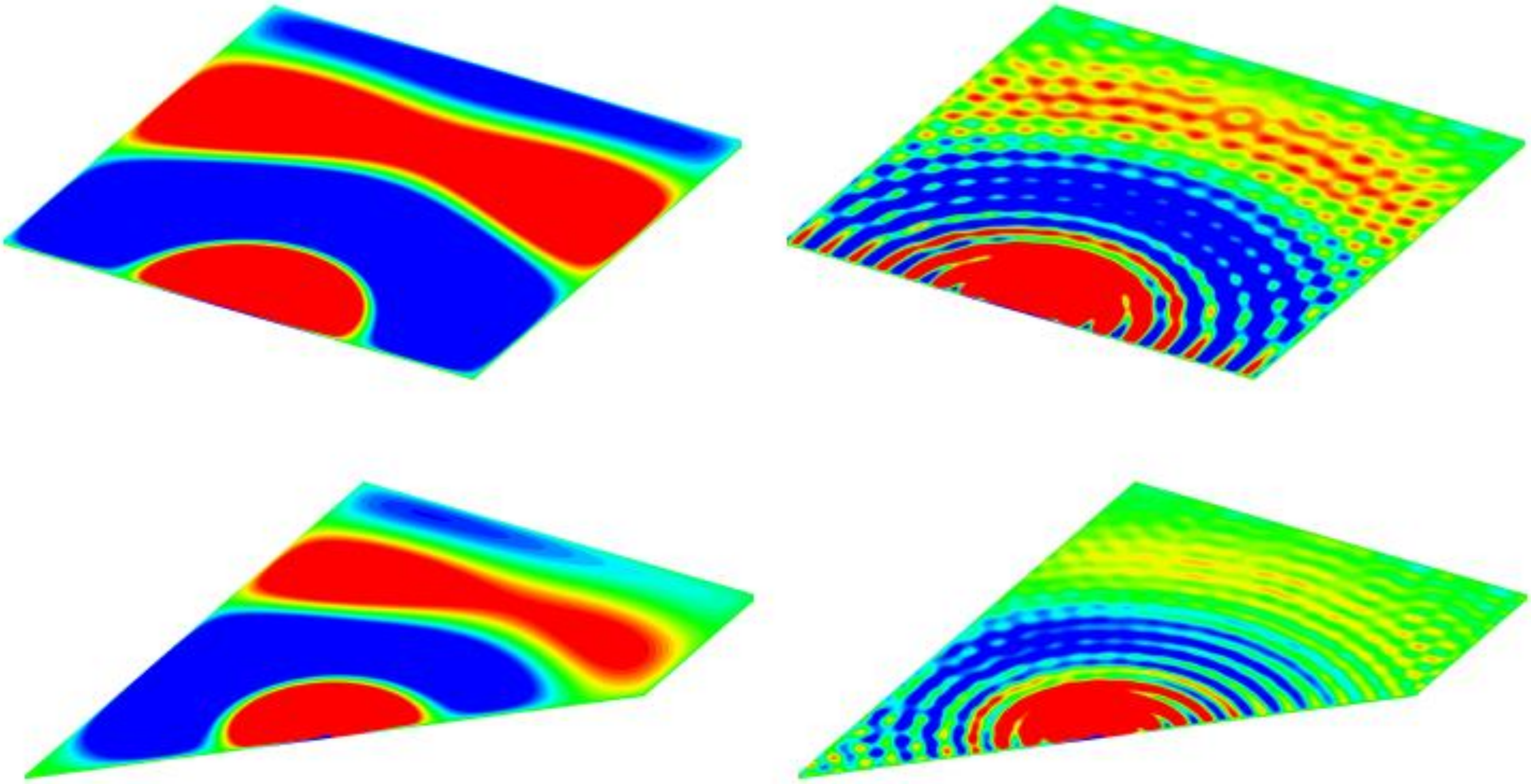


Poro-Elastic Trailing Edges



Acoustic scattering by poro-elastic plates with arbitrary geometry
Pimenta et al., JCP *in preparation*

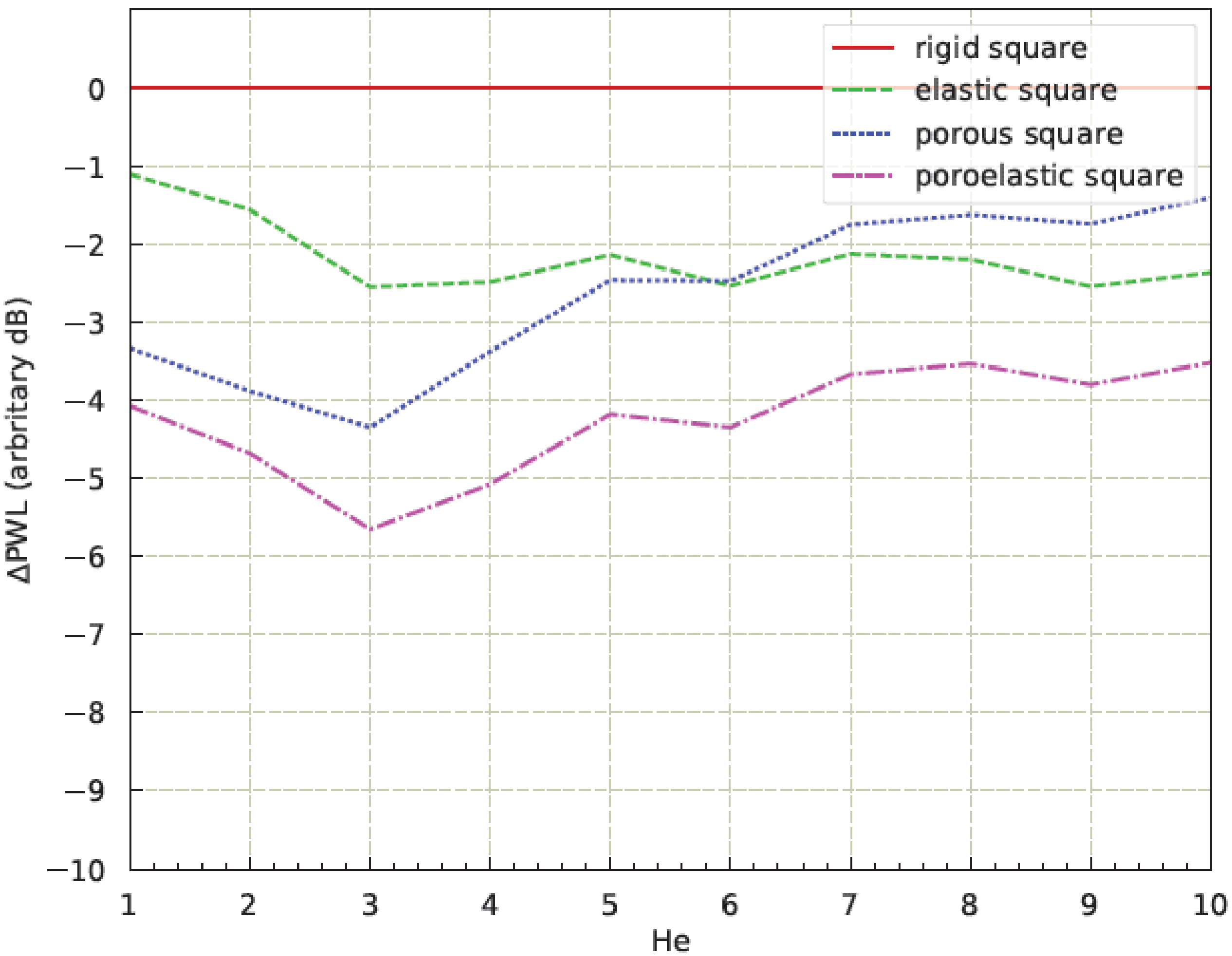
Poro-Elastic Trailing Edges



Destructive interference effects of acoustic surface pressure combined with trailing edge sweep reduce far-field noise

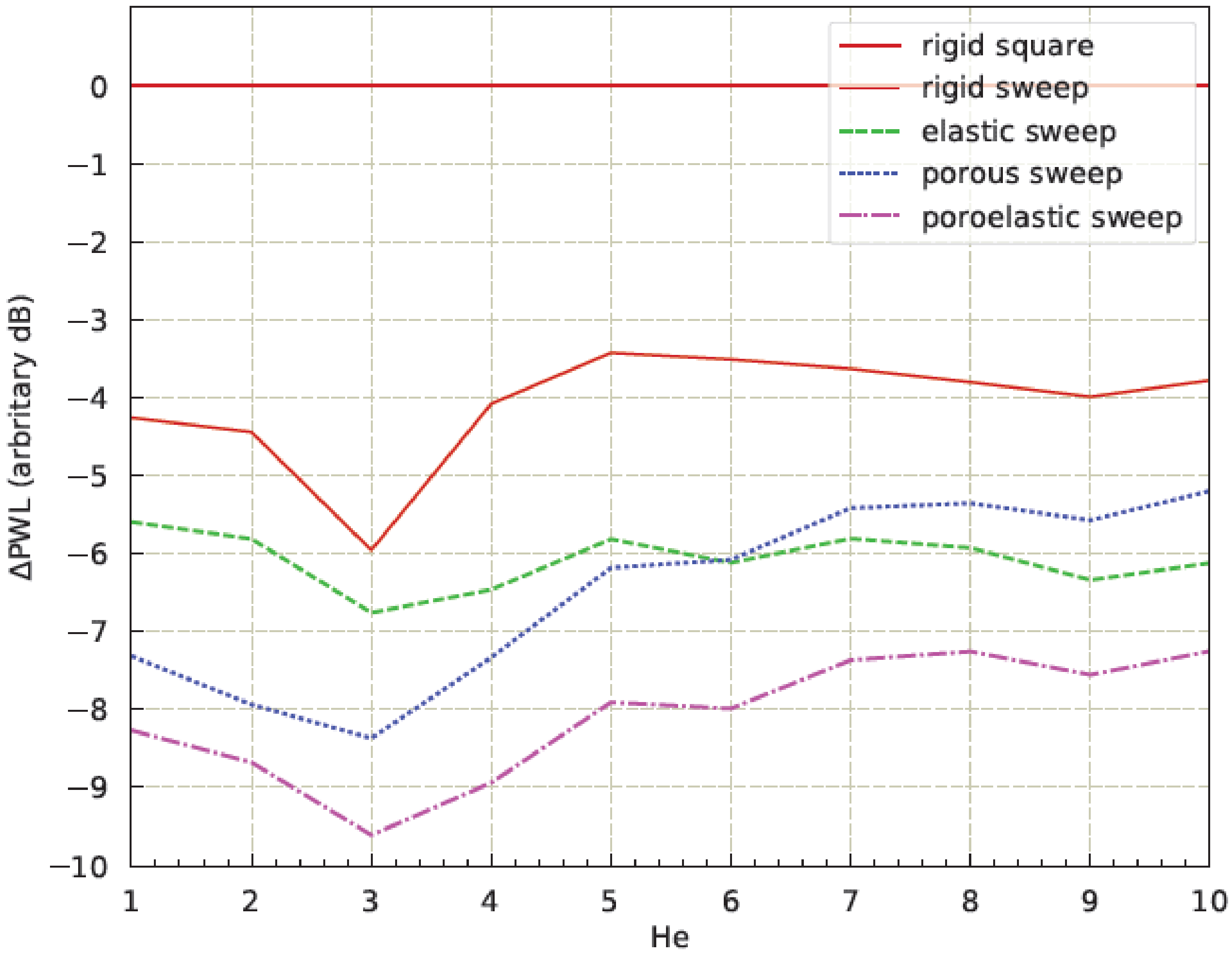
Poro-Elastic Trailing Edges

Sound power level in the far-field



Poro-Elastic Trailing Edges

Sound power level in the far-field



Acknowledgements

FAPESP

BOEING

U.S.ARMY

PRP pró-reitoria de pesquisa unicamp

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

Motivation for Future Work?

Thank you!!!